STATENS GEOTEKNISKA INSTITUT
SWEDISH GEOTECHNICAL INSTITUTE

Prediction of settlements of embankments on soft, fine-grained soils.

Calculation of settlements and their course with time.

ROLF LARSSON
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Preface

This publication contains a recommendation for the way in which prediction of settlements of embankment fills on soft, fine-grained soils should be performed, together with part of the background for the described method. The method is limited to calculation of one-dimensional consolidation.

Calculation of settlements and their course with time for embankments on soft fine-grained soils, primarily clay, has been a major area for research and development at the Swedish Geotechnical Institute (SGI) since the Institute was founded in 1944. The first instrumented test embankment was constructed in 1945, and this and several other more recent fills and instrumented parts of road embankments have been studied since then. At the same time, new techniques and equipment for field testing, instrumentation and sampling as well as testing in the laboratory have been developed.

The results from these investigations, together with experience from consulting activities and from other institutions, have provided a good insight into the processes taking place in soils during consolidation for applied loads.

In parallel with this, new calculation models and techniques have been developed, which enable an accurate prediction of the consolidation processes also in complicated soil profiles and loading sequences.

The research results have been continuously reported in the Institute's series of reports and in international publications and conference proceedings. They have also been continuously incorporated in the work and calculation practices of the institute.

This publication is a digest of the technique for investigations and calculations recommended for prediction of settlements of embankments.

Information 13 is addressed to those who perform or use this kind of prediction. It also describes the background to the EMBANKCO computer programme, which has been developed for this purpose in co-operation between the Swedish National Road Administration and SGI. Paragraphs that are directly related to this programme have been marked specially in the text.

The publication has been written by Rolf Larsson, Per-Evert Bengtsson and Leif Eriksson at the SGI.

Contents

Chapter 1.
Introduction

The prediction of settlements and their course with time is a central problem in geotechnical engineering. Many structures that are built are very sensitive to settlements and calculations are then made in order to check that the settlements are not becoming so large that reinforcing measures such as piling or installation of lime/cement columns are necessary. For other constructions, such as road and railway embankments, large settlements can often be accepted provided that the settlements are even and that their course with time can be predicted. The demand for even settlements then applies both along and across the embankments and is fulfilled by adjustment of the geometry of the embankment and the density of the embankment fill material with regard to the properties of the underlying soil.

A good prediction of the consolidation process is required also in cases where the settlement of the embankment is to be taken out in advance by preloading, with or without the use of vertical drains, and when the embankment for stability reasons is to be constructed in stages to utilize the increase in strength during consolidation.

DETAILED KNOWLEDGE OF THE SOIL PROFILE

A good prediction of the settlements and their course with time demands a detailed knowledge of the underlying soil profile concerning stratification and compression characteristics of the different soil layers and also the in situ stress situation and the pore pressures, as well as the natural variation of the latter. Furthermore, knowledge is required about the draining properties in different seams, varves and layers within the compressible soil as well as in the dry crust and the underlying firmer bottom layers.

In soft soils with low permeability, such as clayey silt, clay and gyttja, the consolidation process is very slow and normally the consolidation properties cannot therefore for practical reasons be determined by in situ tests. Undisturbed samples then have to be taken and the consolidation properties determined by oedometer tests in the laboratory.

CLASSICAL ONE-DIMENSIONAL THEORY

The results from tests on small specimens in the laboratory must then be transferred to in-situ conditions in the settlement prediction. In this process, very simple assumptions have previously been employed. In the classical one-dimensional consolidation theory, developed by Terzaghi in 1923 [1], it is assumed that the relation between applied stress and compression after consolidation, i.e. when all excess pore pressures created by the load increase have dissipated, is a unique relation which is independent of time and drainage conditions. In calculation of the consolidation process, it is also assumed that both the compression modulus and permeability are constants throughout the process without regard to the changing stress level and that the pores in the soil are compressed.

The coarseness of the assumption of a unique relation between applied load and final compression independent of time can readily be observed in laboratory tests on high-plastic clays which exhibit large creep effects. Numerous observations have also been made in the field of settlements that continue in spite of the fact that all excess pore pressures have dissipated. Long-term observations in the field also show settlements and pore pressures that widely deviate from the classical consolidation theory both regarding their size and their course with time. This is also valid in cases which have taken into account changes in modulus related to the stress level alone and changes in permeability during the consolidation process.

TIME-DEPENDENT COMPRESSIBILITY

The time-dependent relation between effective stress and compression has been given many names, such as creep, secondary compression, time resistance and rate effects, all of which are used to describe the same phenomenon. Many different approaches have been used to take these time effects into consideration in the calculation of settlements. In the simplest approach, classical consolidation theory has been assumed to be valid until all excess pore pressures have dissipated, whereupon secondary compression has commenced. This latter compression has then proceeded at a continuously decreasing rate, because secondary compression is assumed to be a linear function versus the logarithm of time. This division has appeared to be applicable to the results of small scale oedometer tests but is inconsistent with long-term observations in the field.

Taylor's laboratory model

The first theory where time effects were at least partly incorporated in the consolidation processes was presented by Taylor in 1940 and was followed two years later by a model for a general variation of compression with effective stress and time [2,3]. The model was mainly valid for results from oedometer tests.

Suklje's general model

A more general model was presented by Suklje in 1957 [4]. In this model, no separation is made between the processes during pore pressure dissipation and thereafter, except for the hydrodynamic delay of the settlement process during the first and more rapid phases of the consolidation process. In the model, the relation between effective stress and compression changes continuously with the rate of deformation.

The so called "secondary consolidation" is thus not a separate phenomenon which is added after a "primary consolidation". On the contrary, the entire course of stress-deformation is time dependent. The development of this time dependent process is further regulated by the permeability of the soil and the drainage paths since a corresponding volume of water has to flow out from the soil in order to allow the settlements to occur.

Bjerrum's geological model

Another presentation with a corresponding model was made by Bjerrum (1967 and 1972) with the purpose of illustrating the influence of time on settlements under loads that act for a long time [5,6]. The presentation was made in order to explain apparent overconsolidation effects in natural soils because of geological "ageing" and to explain why settlements may occur under structures in spite of the fact that the apparent preconsolidation pressure has not been exceeded.

Bjerrum´s presentation is well suited for illustrating processes that continue for a very long time, where the influence of the hydrodynamic delay of the settlement process has been equalised. In the shorter time perspectives that are considered in normal prediction of settlements, the influence of the permeability and the drainage paths has to be taken into account. This was also pointed out by Bjerrum, but is often not evident in later presentations of the model.

Bjerrum's model also includes time-dependent increase in shear strength, because the relation between the undrained shear strength and the apparent preconsolidation pressure is assumed to remain the same also when the apparent preconsolidation pressure is increased by time-dependent settlements at constant load.

The two latter models have been shown to agree very well with the observations of the consolidation processes that have been made both in the field and in the laboratory.

EMPIRICAL KNOWLEDGE

In calculation of settlements, a relevant model is required of what occurs in the soil during the consolidation process apart from the vertical compression. Empirical knowledge about this has been obtained from a large number of field tests, in which the pore pressure build-up during loading and the pore pressure dissipation thereafter have been studied. Furthermore, the elastic deformations during loading and the horizontal movements occurring thereafter have been studied, together with the distribution of settlements with depth in the soil profile. In some cases, also the shear strength increase has been studied [7].

In the laboratory, empirical knowledge has been gathered concerning the compression and swelling characteristics of different soils together with the time dependence of these properties and the relation between compression and permeability. Simultaneously, new test methods and equipment have been developed, enabling a careful determination of the compression characteristics of the soil and its permeability as a function of the compression. From observations in the field, it has also been possible to calibrate the apparent preconsolidation pressures evaluated from the laboratory tests against those stresses in the field at which the behaviour of the soil is significantly changed, [8,9]. It has also been possible to calibrate the initial permeability determined in the laboratory against field tests.

In this way, it has been possible to create a method for transferring the results from small scale oedometer tests in the laboratory to the conditions that apply in full scale loading cases in the field.

SETTLEMENT CALCULATIONS

In calculation of settlements, the soil between two draining layers has often been simplified to a single homogeneous layer. This has been done in order to facilitate hand calculations but involves considerable errors. Alternatively, the soil has been divided into a limited number of layers and the calculation of the consolidation process has been performed using a graphical procedure. Using the latter method also potentially makes it possible to take into account varying moduli and permeability as well as time effects. This is done by calculating the consolidation process for a limited time interval with constant parameters. Thereafter, new parameters are selected with regard to the deformation that has occurred and the excess pore pressure is adjusted with regard to the time dependent changes in the compression characteristics that have occurred during the elapsed time. The consolidation process is then calculated for a new limited time step and the procedure is repeated. For practical reasons, the number of layers and time steps in this procedure are limited in calculation by hand.

The calculation of the distribution of stress increase in the soil because of the applied load has also often been simplified in hand calculations. The simplifications have seemingly not involved very large differences in additional stresses but, because of the variability of the soil and its non-linear compressibility, the differences in calculated settlements can be very large.

No need for simplified assumptions

With access to modern calculation aids, such as personal computers with large computational capacity, there is no need for simplified assumptions and calculation procedures. The soil profiles can be divided into a large number of layers and the time steps in the calculations can be made very short. Likewise, the calculation of stress distribution can be performed utilizing all the possibilities provided by the theory of elasticity. This theory is not perfect but is the best of its kind that is available today.

Experience from this type of calculation is that the results agree far better with what in reality occurs in the soil in the field, concerning both settlements and settlement rates and development of pore pressures and increase in undrained shear strength.

Chapter 2.

General requirements on calculations and calculation data

In order to obtain a sufficiently accurate prediction of settlements and their course with time, the following requirements must be fulfilled:

- The stratification of the soil profile and the drainage paths must be known in detail.
- The in situ pore pressures must be measured and the seasonal variation must be prognosticated.
- Undisturbed samples must be taken in all compressible layers to such an extent that the compressibility characteristics of the soil and their variation in different layers and with depth can be established.
- Compression tests in oedometers must be performed to a corresponding extent. The tests can be performed as constant rate of deformation tests (CRS-tests) or as incrementally loaded tests, [10,11]. Alternatively, a combination of both types of tests can be used for a more complete determination of all parameters. ***In both types of tests, it is imperative to follow the Swedish standard concerning both the performance of the test and the interpretation of the results***. Otherwise, the results must not be used in the model and calculation method described in this publication, and a corresponding procedure which is suited to the testing method used has to be applied.
- The calculations must take into account the fact that the compression parameters and the permeability change when the soil is compressed and that the compressibility is time dependent.
- In non-saturated soil, the effect of gas in the pores must be taken into account. This problem is normally negligible in Swedish clays.
- The elastic initial deformations must be calculated.
- In the calculations, the soil must be divided into a large number of layers in order to obtain an acceptable accuracy of the prediction.
- The stress distribution in the soil must be calculated accurately. Simplified methods, such as the 2:1 method, are not accurate enough.
- The loading sequence must be modelled accurately. Any later changes in load, such as adjustments or decreases in effective load because of masses becoming submerged below a ground water table close to the ground surface during the consolidation process, must also be taken into account.

The requirements for accuracy in the settlement predictions for road embankments are mainly governed by the demands on the function of the road specified by the owner of the road. If the requirements listed above are fulfilled, an acceptable agreement between calculated and actual settlements can be expected. However, perfect agreement cannot be expected. This is mainly because of limitations in the amount of investigation that can be performed for each separate case. The calculation model also involves certain simplifications with a separation of the deformations into initial "elastic" deformations followed by one-dimensional vertical consolidation. The agreement is also affected by external factors, for example the load application in the field seldom follows any simple predetermined pattern because of restraints and problems at the site. Seasonal variations in the climate have also been shown to affect the course of the settlements. The influence of these external factors diminishes with time and normally they do not have any practical significance for the long term settlement process.

Chapter 3.
Investigations and sampling in the field

The first condition for accurate prediction of settlements is that the underlying soil must be thoroughly investigated concerning stratification of the soil profile and properties of the different soil layers. The natural stress situation in terms of overburden pressure and pore water pressure must also be clarified.

The field investigations must yield a detailed picture of the stratification in the soil down to firm bottom both across and along the projected embankment. A classification of the different soil layers must be obtained with respect to soil type, compressibility and drainage properties. A possible occurrence of thin draining layers must also be registered together with prevailing pore pressure conditions.

SOUNDING

The soil stratification is investigated by different sounding methods. Previously, weight sounding tests or total pressure soundings were often used to determine the stratification in compressible soils. The soil classification then demanded that at least disturbed samples were taken in the different layers. Thinner layers could not be registered by these methods and also the registered stratification in terms of thicker layers was relatively coarse. The possibility of efficient determination of the required parameters has been increased considerably by the introduction of the cone penetration test CPT with simultaneous measurement of the pore pressures and the dilatometer test [12.13]. The former enables, a very good determination of the stratification of the soil and the occurrence of draining layers or seams. Furthermore, the in situ pore pressure can be determined efficiently in permeable layers and the need for further pore pressure measurements can be minimized.

By using parallel dilatometer tests, an improved soil classification and estimate of the soil density can be obtained together with an improved estimation of the strength properties and stress conditions in the soil profile. Furthermore, a good determination of the compressibility of layers of coarse silt and sand can be made.

In layered soil profiles and other profiles containing overconsolidated soil and silt and sand layers, the required compression parameters for these zones and layers can thus often be obtained efficiently by using modern in situ methods. Elastic properties and compressibility of overconsolidated cohesive soils are normally estimated by using empirical relations with the undrained shear strength, and a measure of the latter property is obtained from both CPT and dilatometer tests. Dilatometer tests in coarse silt and sand yield moduli which can be used in the same way as the oedometer moduli from laboratory tests. An "oedometer modulus" can also be estimated from the results of CPT tests, although its practical use is restricted to normally consolidated sand.

The need for supplementary sampling can thereby be minimized and in principle be limited to layers consisting of clayey silt, clay, gyttja and possibly peat in which undisturbed samples have to be taken for testing in the laboratory.

PORE PRESSURE MEASUREMENT

For registration of thinner seams and layers, soundings with measurement of the sounding pore pressure are required. The pore pressure soundings can also be used for measurement of the in situ pore pressures by halting the penetration in permeable layers and allowing the excess pore pressure to dissipate. A registration of the pore pressure dissipation versus time during such temporary stops can also be used for a coarse estimate of the draining properties of the layer.

Pore pressure measurements must be performed by the installation of piezometers at different levels in the profile. The extent of these installations depends on factors such as the extent to which equalized pore pressures have been measured in the soundings. The measured pore pressures must be linked to nearby observation points in the national ground water network for an estimation of normal ground water conditions and possible variations.

The piezometers can also be used in tests for determination of the in situ permeability as a complement to the oedometer tests in the laboratory.

SAMPLING

Sampling in soft, fine grained soils is performed with a sampler yielding undisturbed samples of high quality. In Sweden, the Swedish standard piston sampler is used, except in peat where a special peat sampler is used [14].

Repeated investigations have shown that high quality samples can be obtained in most compressible soils with the Swedish standard piston sampler. A condition for this is that the recommended sampling procedure is followed [15]. This involves, among other things, that the punching stroke in the sampling operation is performed evenly and very slowly, that a waiting time of 5 to 10 minutes is allowed between the cutting of the sample and the start of extraction of the sampler and that the extraction is performed slowly and smoothly. All vibrations should be avoided and shutters should be applied only in exceptional cases where these have proved necessary in order to obtain samples.

The sampling equipment must be in perfect condition with a sharp and undamaged cutting edge, sampling tubes that are not excessively worn, well functioning mechanisms and no binding parts.
In the St 1 equipment type, the locking of the release band (or rod) and the breaking mechanism must also function properly.

When sampling in profiles with stiffer soil overlying weaker soil, the stiff soil must be pre-drilled. This must always be done through the dry crust and the stiffer soil directly below.

Chapter 4.
Laboratory investigations

Calculations of settlements in cohesive soils are normally based on results from oedometer tests. These can be performed as incrementally loaded tests or as continuously loaded CRS tests, which are nowadays more common.

DETERMINATION OF CONSOLIDATION CHARACTERISTICS BY OEDOMETER TESTS

There are a number of methods of performing and interpreting oedometer tests, which all yield different results. A condition for the results to be applied in the calculation model described in this publication is that the tests must be performed and interpreted in accordance with the recommendations given in Swedish Standard SS 02 71 29 or SS 02 71 26 for incrementally loaded tests and CRS-tests respectively, [10, 11].*)

The parameters required for a settlement calculation are primarily:

Compressibility parameters:

M_0 = Constant modulus at stresses below the apparent preconsolidation pressure

σ'_c = Apparent preconsolidation pressure

M_L = Constant modulus for stresses just above the apparent preconsolidation pressure

σ'_L = Limit pressure (> σ'_c) at which the modulus starts to increase

M' = Modulus number, $\Delta M / \Delta\sigma'$ for stresses larger than σ'_L

Permeability parameters:

k_i = initial (natural) permeability

β_k = coefficient of change in permeability with compression ($= -\Delta\log k / \Delta\varepsilon$)

Creep parameters:

$\alpha_{s(max)}$ = coefficient of secondary compression ($=\Delta\varepsilon / \Delta\log t$) at the apparent preconsolidation pressure

β_{α_s} = coefficient of change in coefficient of secondary compression with compression, ($\Delta\alpha_s / \Delta\varepsilon$)

For more complicated loading cases involving both loading and unloading, there is also a need for parameters describing the swelling properties of the soil.

The most efficient method of determining the compression characteristics is the CRS-test, which yields continuous relations for the compressibility and permeability properties. Creep parameters and swelling characteristics can normally be estimated with sufficient accuracy by using empirical relations.

In the normal test procedure, the modulus M_0 is estimated from empirical relations for both types of oedometer tests. In those cases where a more accurate determination of the modulus at low stresses (< σ'_c) is required, incremental tests with loading, unloading and reloading are performed according to the standard.

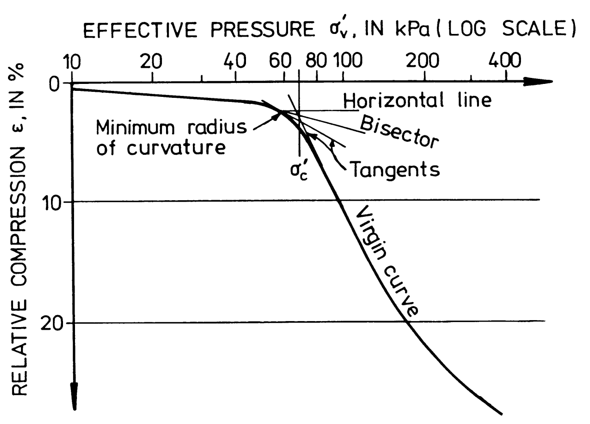
When this is necessary, more accurate determinations of the creep parameters and the swelling characteristics can also be obtained by using incremental oedometer tests.

For peat, incrementally loaded oedometer tests are normally performed, [14].

The compression characteristics evaluated from oedometer tests performed and evaluated according to the Swedish standard are representative for a rate of relative compression of $\alpha_s \cdot 5 \cdot 10^{-6}$ 1/s. This rate is a parameter that is used in the subsequent calculations where the time (or rate) dependence of the compression characteristics is taken into account [7].

*) Oedometer tests according to the Swedish standard are performed on 20 mm high specimens. Incrementally loaded tests are performed with a doubled load in each increment and the load increments are applied for 24 hours. Drainage is provided at both ends of the sample. CRS-tests are performed with a constant rate of deformation and with drainage at one end and pore pressure measurement at the other end. The rate is adapted to the type of soil in such a way that the pore pressure at the undrained end never exceeds 15 % of the total applied vertical stress.

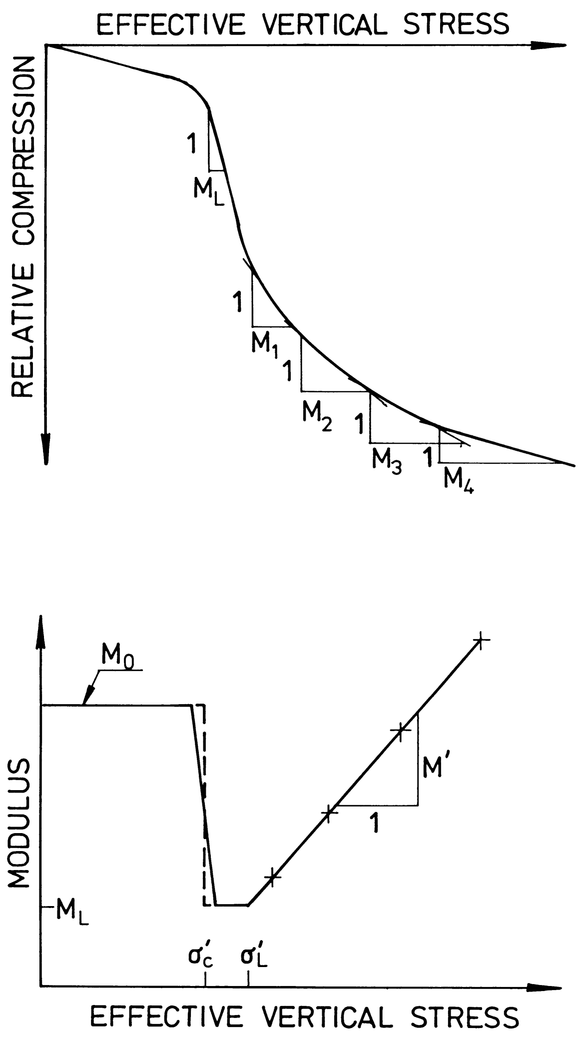
- For ***incrementally loaded tests***, the apparent preconsolidation pressure is evaluated by plotting the 24-hour readings of strain versus the logarithm of the applied stress and using the Casagrande construction.



Determination of the apparent preconsolidation pressure according to Casagrande.

The stress-strain relation is also plotted with the stress in a linear scale in order to verify that there is a significant break in the stress-strain relation at the apparent preconsolidation pressure and to enable an evaluation of the compression modulus. The modulus is evaluated as a tangent modulus at the straight portion of the stress-strain relation just after passing the apparent preconsolidation pressure, M_L, and at a number of points along the curved part of the relation at higher stresses.

The evaluated moduli are then plotted versus the effective stress at which they were evaluated and the modulus number $M' = \Delta M / \Delta\sigma'$ is evaluated from the straight line relation for high stresses.



Evaluation of compression mudulus from oedometer tests

The limit stress σ'_L , at which the modulus starts to increase, is calculated by fitting in such a way that the equation

$$\varepsilon = \frac{\sigma'_L - \sigma'_c}{M_L} + \frac{1}{M'}[\ln(\frac{M'(\sigma' - \sigma'_L)}{M_L} + 1)]$$

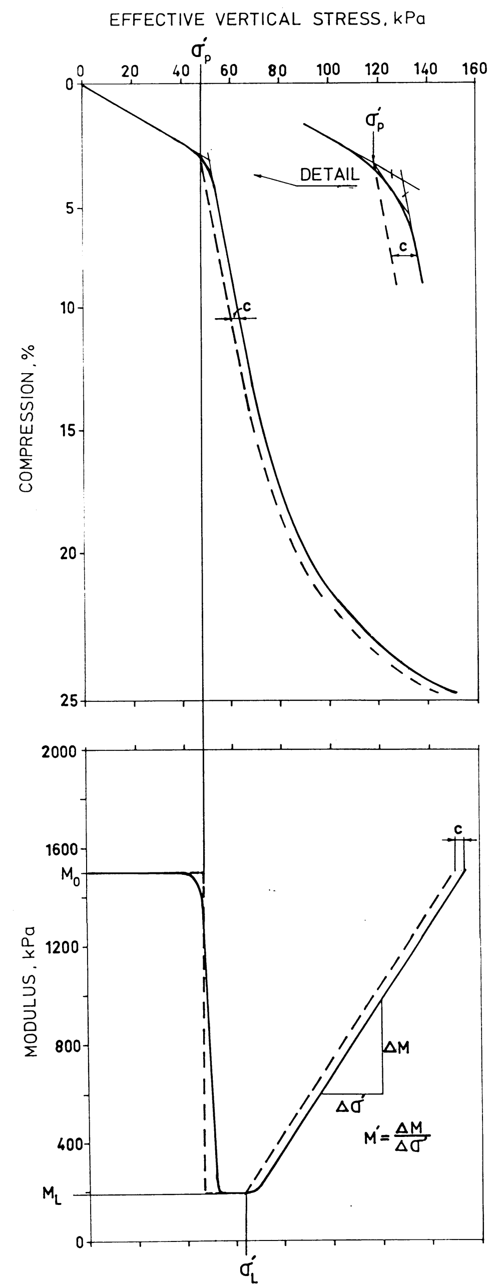
corresponds to the measured stress-strain relation for stresses higher than the apparent preconsolidation pressure.

- In **CRS-tests**, the effective stress, σ′, is calculated from

$$\sigma' = \sigma - \frac{2}{3}u_b$$

where
σ = applied vertical stress
u_b= measured pore pressure at the undrained end

The effective stress is plotted versus the strain in linear scales and the apparent preconsolidation pressure is evaluated by using the Sällfors construction.



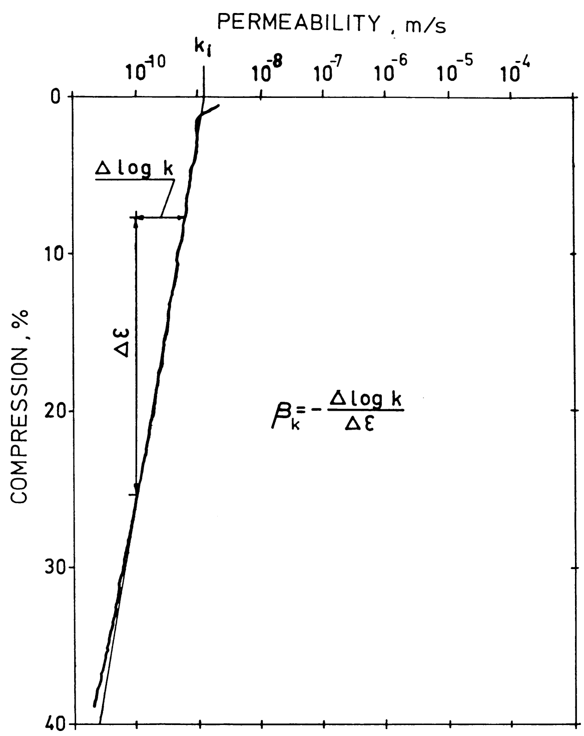
Evaluation of compressibility parameters from the CRS-test

The modulus M_L is evaluated from the straight portion of the stress-strain relation for stresses slightly higher than the apparent preconsolidation pressure and the measured stress strain relation is then corrected by subtracting a factor (c, kPa) from the stress in order to make the straight portion of the curve pass through the point where the apparent preconsolidation pressure was evaluated. This correction is intended to be a correction for rate effects in the test. The modulus-stress relation is evaluated and plotted continuously and the modulus number M′ is evaluated from this plot. The limit pressure σ'_L is then estimated and fitted as above in order to make the evaluated parameters fit the corrected stress-strain relation.

The permeability of the soil is continuously evaluated from

$$k = \frac{\partial \varepsilon}{\partial t} \frac{g \rho_w h^2}{2u_b}$$

The logarithm of the permeability is then plotted versus the relative compression and the initial permeability, k_i, and the coefficient of change in permeability with compression, β_k, are evaluated.



Evaluation of permeability parameters from the CRS-test

CHAPTER 5.
Behaviour of soft soils – empirical experience

5.1 GENERAL

A large number of observations have been made regarding the general behaviour of soil at loading. These observations constitute the basis for the empirical relations used for estimation of creep parameters, swelling characteristics, reloading moduli, moduli of elasticity for calculation of initial deformations and generation of pore pressure changes at changes in the loading situation [7].

5.2 LABORATORY TESTS

Secondary compression

The coefficient of secondary compression, α_s, at an increase of the load has been shown to be a function of the compression of the soil, *Fig. 1*.

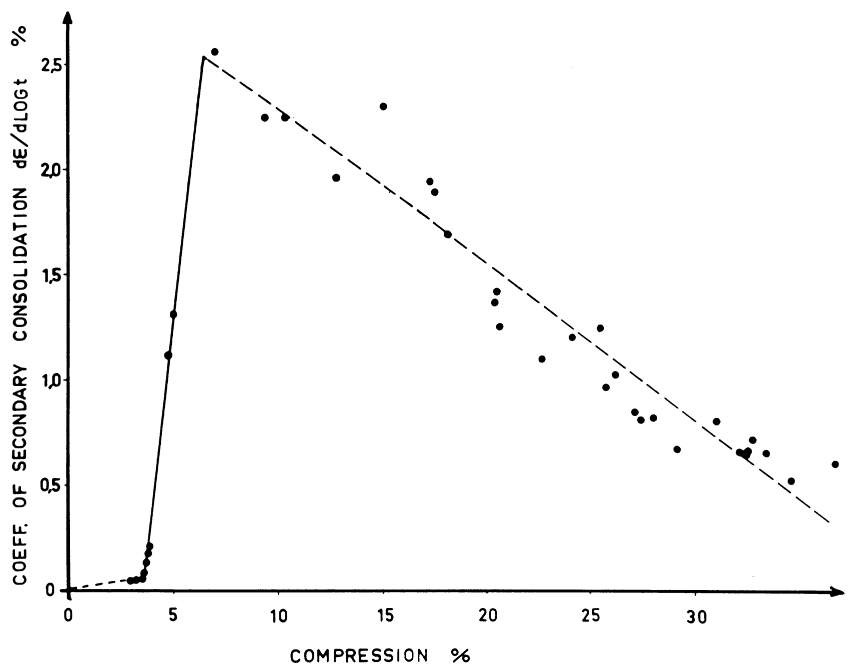


Fig. 1. The coefficient of secondary compression as a function of the compression in Bäckebol clay at 8 m depth [7].

The coefficient has a very low value until a certain compression has been reached, whereupon it rapidly increases to a maximum value and then slowly decreases with further compression. In clay, the critical compression where $_s$ starts to rapidly increase corresponds to an effective stress of about $0.8{\cdot}\sigma'_c$.

The value of the coefficient of secondary compression at its maximum and at further compression is strongly related to the void ratio of the soil, which can also be expressed in terms of soil type and water content. Maximum values of α_s and the related water contents for a number of Swedish soils are shown in *Fig. 2*. The figure shows that there is a clear relation between water content and maximum value of α_s but also that this relation varies between different types of soil. It has also been shown that the decrease in α_s at compression corresponds to the simultaneous decrease in water content and follows the same relation as for maximum values and water content.

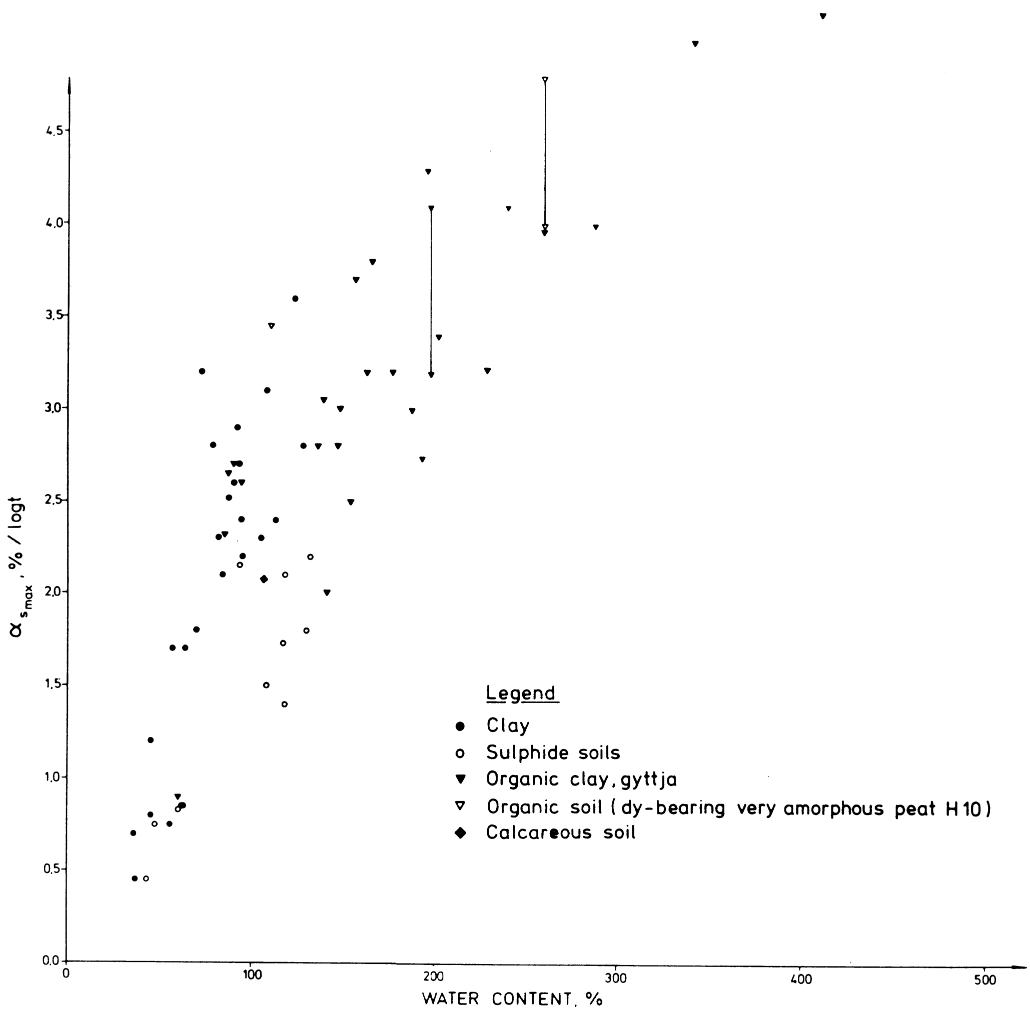


Fig. 2. Relation between maximum coefficient of secondary compression and natural water content in different soils [7].

The variation of a_s at loading above the apparent preconsolidation pressure can be written

$$a_{s\,\max(e)} = a_{s\,\max} - b_{a_s}\Delta e$$

where

$a_{s\,max}$ = maximum value of a_s

$a_{s\,max(e)}$ = a_s at compression *De*

b_{a_s} = coefficient of change in coefficient of secondary compression with compression, $-Da_s / De$

De = relative compression between the actual compression and the compression at $a_{s\,max}$

Both $a_{s\,max}$ and b_{a_s} can be estimated from *Fig. 2*. on the basis of soil type and natural water content.

Values of the coefficient of change in a_s with compression, b_{a_s}, are obtained by a recalculation using the density of solids in the soil and can be estimated from

$$b_{a_s} \approx a_{s\max}(w_N + 0.37)/(w_N - 0.25)$$
(clay and organic clay)

$$b_{a_s} \approx (a_{s\max} - 0.002)(w_N + 0.37)/(w_N - 0.25)$$
(sulphide soil)

$$b_{a_s} \approx (a_{s\max} - 0.0145)(w_N + 0.5)/(w_N - 0.2)$$
(gyttja)

Measured values in Swedish peat indicate that the coefficient of secondary compression in this type of soil can be related to the degree of humification rather than water content and that the parameter b_{a_s} is negligible [17], *Fig. 3*.

As has been shown, the secondary compressions are very small until the effective stresses approach the apparent preconsolidation pressure. In modelling the behaviour of soil, it is therefore very important to consider the stress and deformation history of the soil in situ (primarily the overconsolidation ratio) also in describing the time dependence of the properties.

A table with guiding values has been developed for selection of empirical values of $\alpha_{s\,max}$ and b_{a_s}, Table 1, for use in the EMBANKCO programme.

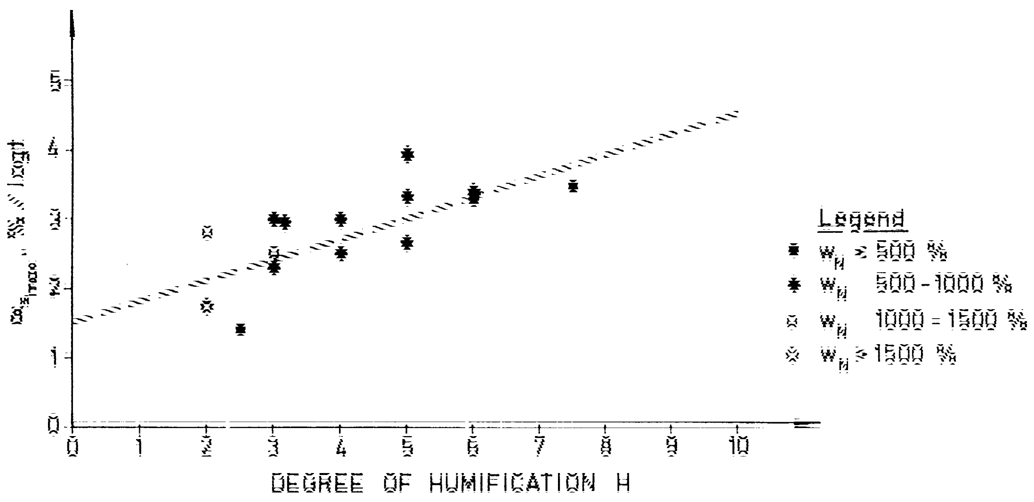


Fig. 3. Maximum coefficient for secondary compression in peat as a function of the degree of humification [17].

Table 1. Table with guiding values for use in EMBANKCO.

Clay and slightly organic clay			Organic clay, gyttja, sulphide clay, calcareous clay		
$w_{N'}$ %	$\alpha_{s\,max}$	β_{as}	$w_{N'}$ %	$\alpha_{s\,max}$	β_{as}
25	0.000	0.000	25	0.000	0.000
30	0.002	0.027	50	0.007	0.030
40	0.006	0.031	75	0.016	0.033
50	0.010	0.035	100	0.021	0.035
60	0.014	0.039	125	0.026	0.038
70	0.018	0.043	150	0.030	0.040
80	0.021	0.046	200	0.036	0.046
90	0.025	0.049	250	0.040	0.051
100	0.029	0.053	300	0.044	0.055
110	0.033	0.057	350	0.047	0.058
120	0.037	0.061	400	0.050	0.061

For peat, $\alpha_{s\,max}$ = 0.025 and β_{α_s} = 0.000 are normally used.

Swelling

When the effective stresses in a soil without cementation are decreased to a sufficient degree, the soil starts to swell. In a clay which has only just consolidated for a stress above the previous apparent preconsolidation pressure (i.e. the excess pore pressures have just dissipated) secondary compressions are in progress at a relatively high rate. A small reduction in load will halt this compression for a certain period of time and possibly there will be a slight swelling before the secondary compression commences at a reduced rate. If the load reduction is large enough, the secondary compression stops permanently and for even larger load reductions the clay starts to exhibit secondary swelling in addition to the "immediate" swelling.

Swelling is time dependent in the same way as compression. At unloading, the pore pressure in the soil decreases. A hydrodynamic delay of the swelling occurs for a certain time, during which the rate of the swelling depends on the permeability of the soil and the access to free water. After equalization of the pore pressure, the secondary swelling proceeds and may be further delayed if the supply of free water is cut off. Apart from the demand for access to free water to be sucked up, the swelling process is in principle identical to the compression process.

Oedometer tests with incremental loading and unloading show that the swelling or unloading modulus is very high for stresses close to the new apparent preconsolidation pressure, but decreases successively with decreasing effective stress and becomes very low at low effective stress.

- **The swelling modulus M_s** can be expressed as

$$M_S = \sigma' / a_S$$

where σ' = effective vertical stress
a_S = swelling index

When a normally consolidated soil is subjected to a small load reduction, the secondary consolidation continues at a reduced rate. If the load reduction is large enough, also the time dependent deformation change direction to secondary swelling. The stress level at which this occurs can be written as $b{\cdot}\sigma'_c$, where b is a load factor. The coefficient of secondary swelling at stresses below $b{\cdot}\sigma'_c$ can apparently be described as $c{\cdot}S$, where c is a constant and S is the total swelling that has developed at the particular time.

Swelling in oedometer tests

The net effect of the processes described above during a 24-hour interval for a load step in the oedometer is that at small load reductions there will be a small compression in spite of the reduced load. The compression at the stress level $b{\cdot}\sigma'_c$ is often almost identical to the compression at the start of the unloading and for larger load reductions the net effect becomes a real swelling, *Fig. 4*.

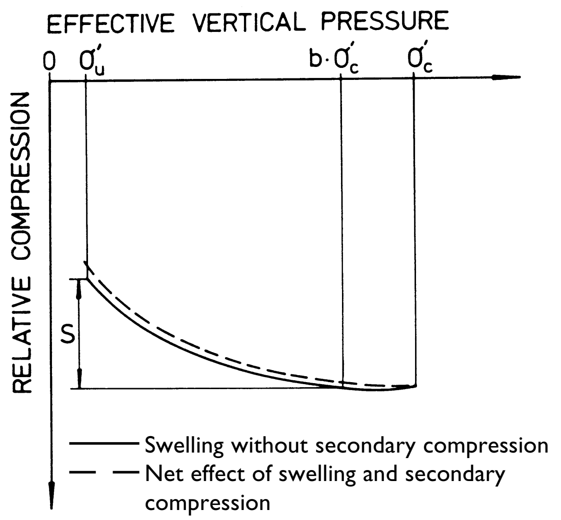


Fig. 4. Deformations at unloading in an oedometer test [7].

According to the results obtained in Swedish soils, the swelling after 24 hours in incremental oedometer tests can be expressed as

$$S = a_S \ln \frac{b \cdot \sigma'_c}{\sigma'_u}$$

where S = total relative increase in specimen height
a_S = swelling index
σ'_c = preconsolidation pressure
b = load factor
σ'_u = stress after load reduction

This equation has been applied to results from tests on a large number of soil types and has been found to correspond well to the results.

For clays, the swelling index a_S has been found to vary between 0.007 and 0.012 and the load factor b to be about 0.8. The index a_S decreases with increasing grain size and becomes about 0.001 in sand and gravel. The factor b increases with increasing grain size to become about 1.0 for gravel. Values of a_S for a number of Swedish soils as a function of average grain size are shown in *Fig. 5*. The upper part of the figure also shows approximate values of the load factor b. Investigations in Swedish peat show that the load factor is about 0.8 also for this type of soil [18].

In clays, there is probably a correlation between the parameters a_S and b and the plasticity of the clay. Preliminary relations of this type are shown in *Fig. 6*.

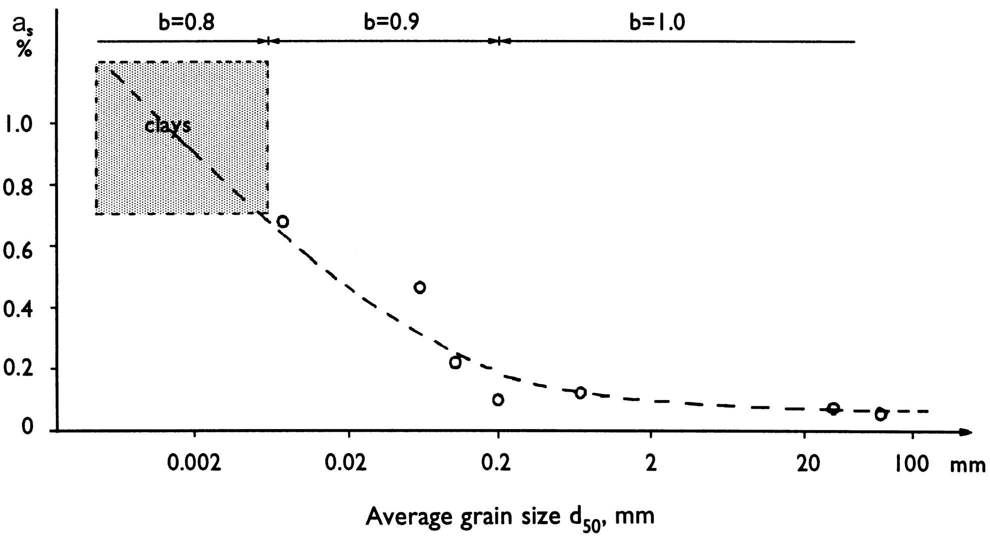


Figure 5. Swelling index a_s as a function of average grain size [7].

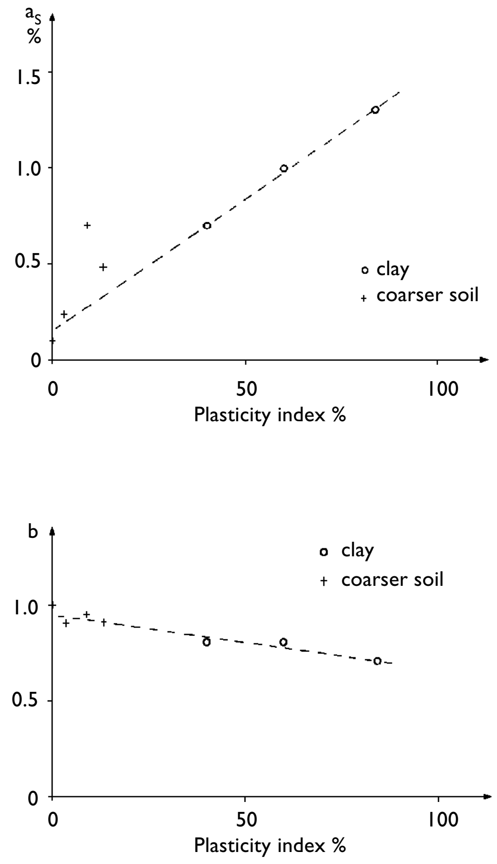


Figure 6. Swelling parameters as a function of plasticity index [7].

Swelling in practical cases of unloading in the field

The method of calculating swelling described above is applicable to the sequence of events in oedometer tests with a specific routine for loading and unloading. The deformations in a practical case in the field depend among other things on how long the soil has been subjected to various loads and the size of the time dependent settlements that has developed before the unloading started, the unloading sequence, how much time dependent swelling has occurred and access to free water.

Calculation of these deformations requires knowledge of how the secondary deformation varies with both stress and strain in the soil. Preliminarily, the coefficient of secondary consolidation α_s can be assumed to decrease linearly with decreasing stress within the interval σ'_c to $b{\cdot}\sigma'_c$ according to

$$\alpha_s = \alpha_{s\,max(\varepsilon)} \frac{\sigma' - b\sigma'_c}{(1-b)\sigma'_c}$$

where

$\alpha_{s\,max(\varepsilon)}$	=	coefficient of secondary consolidation at first loading and current compression
σ'	=	effective vertical stress
b	=	load factor
σ'_c	=	apparent preconsolidation pressure

The secondary swelling at effective stresses lower than $b{\cdot}\sigma'_c$ can be assumed to be governed by

$$\alpha_{s(swelling)} = -c \cdot S$$

where c = a constant
S = total developed swelling

In the EMBANKCO programme, version 1, secondary swelling is not taken into account. The swelling modulus, M_s, is calculated by using a given multiplication factor as a direct function of the effective vertical stress, $M_s = a{\cdot}\sigma'_c$. The factors 100 for clay and organic soil, 200 for silt and 1000 for sand are given as guiding values for selection of the multiplication factor a.

Reloading

The modulus at reloading after an unloading depends on how much swelling has developed during the unloading period. In cases where the unloading has been small, no significant swelling will have occurred and the reloading modulus will be high.

A characteristic feature of specimens which have swelled is that the compression modulus at reloading up to the effective stress $b{\cdot}\sigma'_c$ can be approximated to a constant. The compression when this effective stress has been reached will be the same as the previous maximum compression. b is the same load factor which also signifies when the specimen starts to swell.

- **The recompression modulus M_{rl}** within the stress interval σ'_u to $b{\cdot}\sigma'_c$ can be written

$$M_{rl} = \frac{b \cdot \sigma'_c - \sigma'_u}{a_s \cdot \ln \dfrac{b \cdot \sigma'_c}{\sigma'_u}}$$

where σ'_u is the effective vertical stress to which the specimen has been unloaded and a_s and b are the swelling index and load factor respectively. The application of this formula is also limited to what happens in incrementally loaded oedometer tests performed according to the standard procedure.

More generally, the equation can be written

$$M_{rl} = \frac{b \cdot \sigma'_c - \sigma'_u}{S}$$

where S is the total swelling strain that has developed during the unloading.

The reloading modulus in soil which has been subjected to only a small unloading to a stress higher than $b \cdot \sigma'_c$ is difficult to measure because of the creep deformations taking place simultaneously. On the assumption that the soil in principle behaves in the same way as at larger unloading, the modulus can in these cases be written

$$M_{rl} \approx \frac{\sigma'_u}{a_s} \qquad \sigma'_u \geq b \cdot \sigma'_c$$

The recompression modulus which is calculated without regard to secondary swelling, apart from what occurs during 24 hours in the oedometer tests, becomes a function of overconsolidation ratio, swelling index and load factor. Recompression moduli calculated by using typical values for a high-plastic clay are shown in *Fig. 7*.

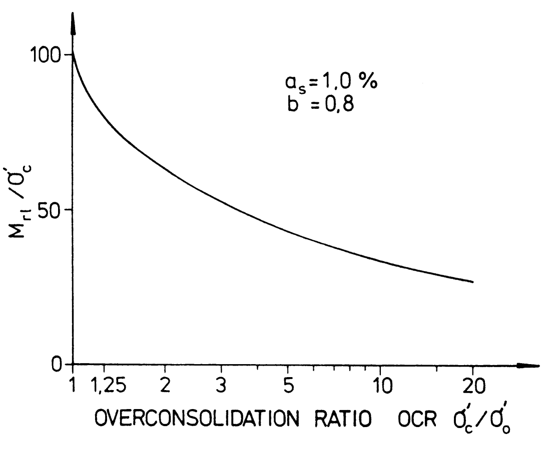


Figure 7. Calculated recompression modulus in high-plastic clay as a function of the overconsolidation ratio [7].

As is evident from the figure, the frequently used empirical values for the recompression modulus of $50 \cdot \sigma'_c$ (or $250 \cdot \tau_{fu}$) are only coarse approximations over a wide span.

The time effects during reloading vary. The secondary swelling does not stop altogether as soon as a small increase in load is applied, but continues within a certain stress interval at a reduced rate. This may to some degree affect the net result of a small increase in load in such a way that no compression, or even a minor swelling, will be the final result.

At further reloading, there will not be any significant creep effects until the effective vertical stress reaches $b \cdot \sigma'_c$. At this stress level and the related compression, the coefficient of secondary compression starts to increase rapidly, which is in concordance with the sequence of events at first loading in the oedometer, until a maximum value is reached which is equal to the value obtained at first loading at the corresponding compression.

In the EMBANKCO programme the compression modulus M_0 for loading up to the apparent preconsolidation pressure is given as input.

In soft, normally consolidated clay, where the influence of this modulus on the total compression is marginal, the modulus is often estimated from more approximate empirical correlations of the type mentioned above

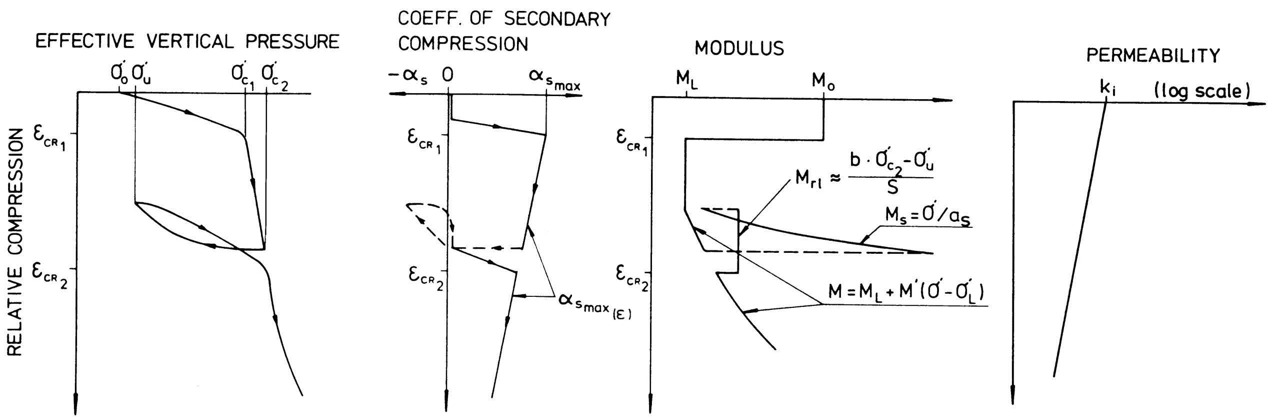
$$\begin{aligned} M_0 &\approx 150 \cdot \tau_{fu} \text{ for organic soil} \\ &\approx 250 \cdot \tau_{fu} \text{ for high-plastic clay} \\ &\approx 500 \cdot \tau_{fu} \text{ for low-plastic clay} \\ &\approx 1000 \cdot \tau_{fu} \text{ for very silty clay/clayey silt} \end{aligned}$$

In cases where this initial modulus has a larger influence on the results of the calculations, more relevant moduli can be estimated from the effective stress in situ, the apparent preconsolidation pressure and the relations presented above.

In cases where the calculations wholly or largely concern compressions at stresses below the apparent preconsolidation pressure, the modulus M_0 should be determined by oedometer tests with unloading from σ'_c to σ'_0 and reloading according to the standard procedure.

General model for moduli and the time dependence of the compressibility

A general model for variation of modulus and coefficient of secondary consolidation during the loading, unloading and reloading sequences is outlined in *Fig. 8 and Table 2*.



MODE	CONDITION	STRESS INTERVAL	MODULUS	COEFF. OF SECONDARY CONSOLIDATION
Loading	$\sigma'_o < b\sigma'_{c_1}$	$\sigma'_o - b\sigma'_{c_1}$	M_o	Negligible
		$b\sigma'_{c_1} - \sigma'_{c_1}$		$\alpha_{s_{max}} \cdot \frac{\varepsilon - \varepsilon_{b\sigma'_{c_1}}}{\sigma'_{c_1}(1-b)/M_o} \leq \alpha_{s_{max}(\varepsilon)}$
	$\sigma'_o > b\sigma'_{c_1}$	$\sigma'_o - \sigma'_{c_1}$		$\alpha_{c_{max}} \frac{\varepsilon}{(\sigma'_{c_1} - \sigma'_o)/M_o} \leq \alpha_{s_{max}(\varepsilon)}$
		$\sigma'_{c_1} - \sigma'_L$	M_L	$\alpha_{s_{max}(\varepsilon)} = \alpha_{s_{max}} - \beta_{\alpha_s}(\varepsilon - \varepsilon_{CR_1})$
		$\sigma' > \sigma'_L$	$M_L + M'(\sigma' - \sigma'_L)$	
Unloading		$\sigma'_{c_2} - b\sigma'_{c_2}$	$M_s = \sigma'/a_s$	$\alpha_{s_{max}(\varepsilon\sigma'_{c_2})} \cdot \frac{\sigma' - b\sigma'_{c_2}}{(1-b)\sigma'_{c_2}}$
		$< b\sigma'_{c_2}$		$-c \cdot S$
Reloading	$\sigma'_u < b \cdot \sigma'_{c_2}$	Eventual small interval	$M_{rl} \approx (b\sigma'_{c_2} - \sigma'_u)/S$	$f(-c \cdot S)$
		$\sim \sigma'_u - b\sigma'_{c_2}$		Negligible
		$b\sigma'_{c_2} - \sigma'_{c_2}$		$\alpha_{s_{max}(\varepsilon_{CR_2})} \cdot \frac{\varepsilon - \varepsilon_{b\sigma'_{c_2}}}{\sigma'_{c_2}(1-b)/M_{rl}} \leq \alpha_{s_{max}(\varepsilon)}$
	$\sigma'_u > b \cdot \sigma'_{c_2}$	$\sigma'_u - \sigma'_{c_2}$	$M_{rl} = \sigma'_u/a_s$	$\alpha_{s_{max}(\varepsilon_{CR_2})} \cdot \frac{\varepsilon - \varepsilon_{\sigma'_u}}{(\sigma'_{c_2} - \sigma'_u)/M_{rl}} \leq \alpha_{s_{max}(\varepsilon)}$

Figure 8 and Table 2. Schematic variation of compressibility (modulus) and time dependence (coefficient of secondary consolidation) at loading, unloading and reloading [7].

The model may at first sight appear very complicated. However, it is only a summary of the empirical observations of the behaviour of soil at loading in the oedometer. For determination of the various parameters in the model, it is normally sufficient for oedometer tests to be performed and existing empirical relations utilized. The exception is cases where the secondary swelling has to be considered, because no empirical experience of this property is available.

- **The critical deformation** ε_{CR1}, where the creep rate attains its maximum value, is calculated from

$$\varepsilon_{CR1} = \frac{\sigma'_{c1} - \sigma'_0}{M_0}$$

where σ'_{c1} is the natural apparent preconsolidation pressure in situ. σ'_{c2} is the new apparent preconsolidation which is created if the compression ε_{CR1} is exceeded. σ'_{c2} is the effective vertical stress on the oedometer curve for the first loading that corresponds to the largest compression that has occurred. σ'_{c2} thus increases with the deformations that occur at secondary consolidation and is a so-called "quasi-preconsolidation pressure" which is related to maximum compression rather than maximum effective vertical stress.

At unloading and renewed loading, the apparent preconsolidation pressure increases as soon as the total compression because of the new loading exceeds the swelling that occurred as a result of the unloading. The apparent preconsolidation can in this way increase during cyclic loading and as a result of time effects, even if the previous maximum effective vertical stress is not exceeded.

The deformation ε_{CR2}, where the coefficient of secondary consolidation after unloading and renewed loading returns to the maximum values at the particular deformation can be calculated from

$$\varepsilon_{CR2} = \varepsilon_{b\cdot\sigma'_{c2}} + \sigma'_{c2}(1-b)/M_{rl} \quad when \quad \sigma'_u < b\cdot\sigma'_{c2}$$

and

$$\varepsilon_{CR2} = \varepsilon_{\sigma'_u} + (\sigma'_{c2} - \sigma'_u)\cdot a_s / \sigma'_u \quad when \quad \sigma'_u \geq b\cdot\sigma'_{c2}$$

All secondary compressions that occur at a higher rate of strain than $\alpha_s \cdot 5\cdot 10^{-6}$ 1/s are incorporated in the compressions calculated by using the different moduli except for M_s in the stress interval (σ'_{c2} - $b\cdot\sigma'_{c2}$) and M_{rl} in the first reloading stress interval where secondary swelling may occur.

For slower processes, the effects of secondary deformations must be taken into account. In those stress intervals where the deformations from moduli and the secondary deformations occur in opposite directions, the deformations from moduli can be regarded as immediate apart from the hydrodynamic delay. The time dependent process then continues as before the stress change but at a reduced rate.

Simplified model for moduli and the time dependence of the compressibility

In most cases of embankments on clay, the loading entails a large increase in vertical stress. A limited reduction in load can then be obtained during the course of consolidation because some parts of the embankment or the upper soil layers are depressed and become submerged below the ground water table. In some cases, a temporary surcharge is also used in order to speed up the settlement process and to enable a subsequent termination of in the ongoing settlement process by a sufficiently large load reduction. Usually, no significant swelling is expected, and particularly not any secondary swelling.

In calculation of settlements of embankments on soft, normally consolidated soils, the values of the parameters M_0, M_s and M_{rl} are normally also of limited importance. What is most important to consider in the soil model for this type of soil and calculation is

- At unloading, an "elastic" swelling occurs at a high swelling modulus.
- The modulus at loading and reloading is very high at effective vertical stresses below the apparent preconsolidation pressure.
- When the effective vertical stress exceeds $0.8\cdot\sigma'_c$ and the rate of strain is lower than $\alpha_s\cdot 5\cdot 10^{-6}$ 1/s, significant secondary compression will occur.
- The coefficient of secondary compression will increase to a maximum occurring at a compression corresponding to the compression at the prevailing apparent preconsolidation pressure.
- Thereafter, the rate of secondary consolidation decreases with further compression.
- The coefficient of secondary consolidation rapidly decreases if the effective vertical stress decreases and stops altogether at an effective stress level of $0.8\cdot\sigma'_c$. (This must be considered).

In this model, no parameters apart from those determined in ordinary oedometer tests are required. Normally, it is sufficient to perform CRS tests because the time dependence (coefficient of secondary consolidation) and the moduli M_0, M_s and M_{rl}can in most cases be estimated with sufficient accuracy from empirical correlations.

> The EMBANKCO programme, version 1, operates with this simplified model.

Modulus of elasticity

When a load of limited extension is applied to the ground, horizontal deformations and associated settlements occur. These horizontal deformations and related settlements are calculated by using the theory of elasticity, an estimated modulus of elasticity and an assumption of $\nu = 0.5$. This calculation is considerably simplified because $\nu = 0.5$ applies to fully saturated and completely undrained conditions, which in overconsolidated soils is not relevant even during the time for load application. Furthermore, the horizontal deformations are assumed to be immediate and occur directly when the load is applied, whereas in reality they continue during a large part of the consolidation process, even if they relatively rapidly decrease in size and importance with time. The modulus of elasticity E is normally estimated from empirical relations based on the undrained shear strength of the soil. Different values of E have been proposed ranging from $E \approx 80 \cdot \tau_{fu}$ for organic soils to $E \approx 2000 \cdot \tau_{fu}$ for low-plastic clay.

However, the properties of soil materials are not linear-elastic but the stress-deformation curves at shear exhibit more or less hyperbolic shapes. This entails that the higher the degree of the shear strength that is mobilized, the lower the modulus of elasticity becomes. According to laboratory tests, the modulus of elasticity can be expressed as

$$E \approx \frac{\tau_{fu} \cdot 215 \cdot \ln F}{I_P}$$

where τ_{fu} = undrained shear strength of the soil according to corrected vane shear tests and direct simple shear tests
F = calculated factor of safety against failure at undrained shear
I_P = plasticity index in the soil

This formula takes account of the non-linearity of the elastic properties and the type of soil. In principle, it is in agreement with the different proposals that have been made based on experience from earlier field tests and also from recent results obtained from the Institute's test embankments.

The deformations calculated in this way may be assumed to comprise both the initial horizontal deformations and those that will develop later with time.

As a rule, the settlements because of horizontal deformations are small at normal safety against shear failure. However, they increase in high plastic clay and particularly in organic clays, and they increase strongly in all types of soil when the safety against failure is reduced.

Permeability

The permeability of a soil is a function of its void ratio and thereby also of its relative compression. It has been shown that within the range of compression which is of practical interest, the permeability can be described with good accuracy by

$$\log k_\varepsilon = \log k_i - \beta_k \cdot \varepsilon$$

where k_ε = permeability at compression
k_i = initial permeability
β_k = $-\Delta \log k / \Delta \varepsilon$

In homogeneous soft clays, there is normally no significant difference in horizontal and vertical permeability. However, in many cases the clay contains various layers and variations in vertical permeability, which entails different conditions for vertical and horizontal water flow. In soils with pronounced structural anisotropy, i.e. where the uneven particles are mainly oriented in the same direction, there is a difference in permeability with direction also in the small scale. Clays which have been subjected to very high vertical stresses often have a pronounced structural anisotropy.

This is also valid for peat soils with low degrees of humification, where the horizontal permeability is often considerably higher than the vertical. The difference decreases with increasing degree of humification. A higher horizontal permeability has been observed also in gyttja with a clear preferred horizontal orientation of the organic particles.

Even very homogeneous clays have a natural variation in water content and void ratio and thereby also in permeability. Considering the small specimens that are tested in the oedometer tests, a variation in measured permeability of at least ±20 % must be expected. This entails that either a large number of laboratory tests or field tests involving a larger volume of soil ought to be performed. In a homogeneous clay profile, it may be assumed that a sufficient number of oedometer tests at different levels will yield a good picture of the permeability and that the effect of the spread in the results will be evened out.

In layered soils, and particularly when the horizontal permeability is of interest, field tests ought to be performed.

Pore pressure generation at loading

A number of methods have been proposed for calculation of the development of excess pore pressures in a soil mass when the total stresses are changed under undrained conditions. Earlier methods were based on different combinations of responses to changes in mean normal stress and in shear stress.

A more rational approach has become possible by taking critical stresses into consideration. It has been shown clearly that a natural clay has a continuous combination of compressive and shear stresses creating a so-called yield surface, at which the behaviour of the clay changes from being essentially elastic to become elasto-plastic with large plastic deformations, *Fig. 9*.

The shape and size of the yield surface can be estimated from the apparent preconsolidation pressure σ'_c, the coefficient of earth pressure in normally consolidated soil K_{onc} and the effective shear strength parameters c' and ϕ'.

In undrained conditions, the initial development of pore pressure in principle is such that the mean effective normal stress p' remains constant

$$p' = (\sigma'_1 + \sigma'_2 + \sigma'_3) / 3$$

When the increasing stresses reach a point on the yield surface, the development of pore pressure changes and in principle becomes such that the yield surface is not passed but the stress path follows the yield surface until undrained shear failure occurs, *Fig. 10*.

For a point in the soil mass below an embankment, this entails that the increase in pore pressure will be less than the increase in applied vertical stress until the effective vertical stress reaches the apparent preconsolidation pressure. Thereafter, the further increase in pore pressure will be equal to the further increase in vertical stress (Curve A in *Fig. 10*). If the soil has an overconsolidation ratio larger than about 2, the stress path up to failure may be such that critical shear stresses are reached before the effective vertical stress has reached the apparent preconsolidation pressure. In this case, the development of pore pressure changes and may become negative when the effective shear strength parameters are mobilized, (Curve B in *Fig. 10*).

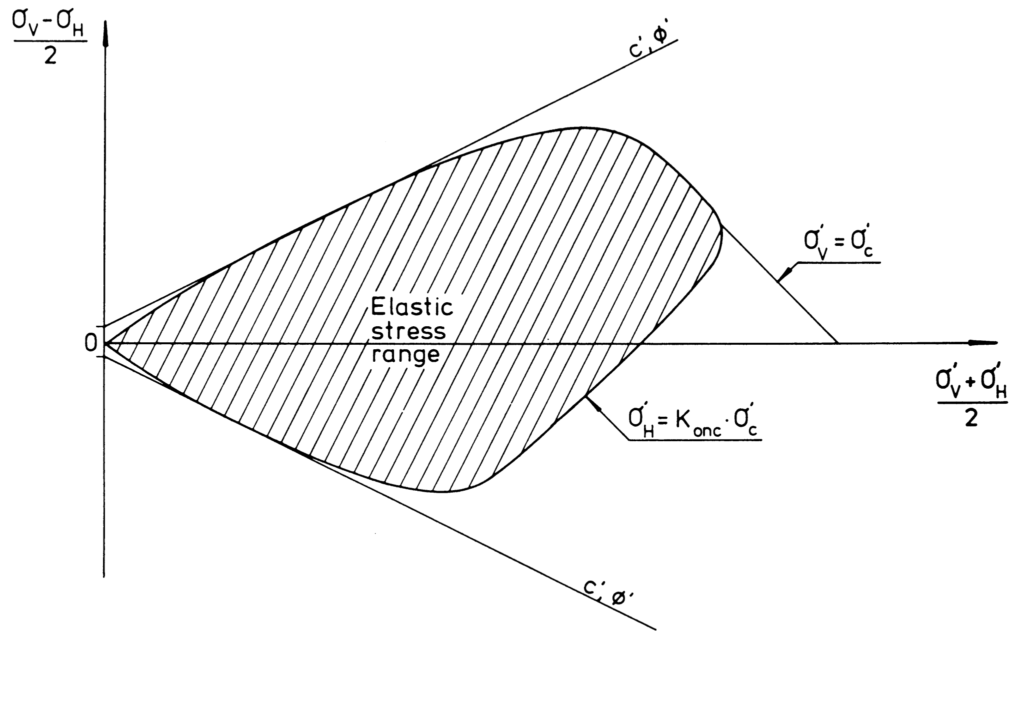


Fig. 9. Schematic yield surface for a soft clay when the principal stresses are vertical and horizontal respectively [7].

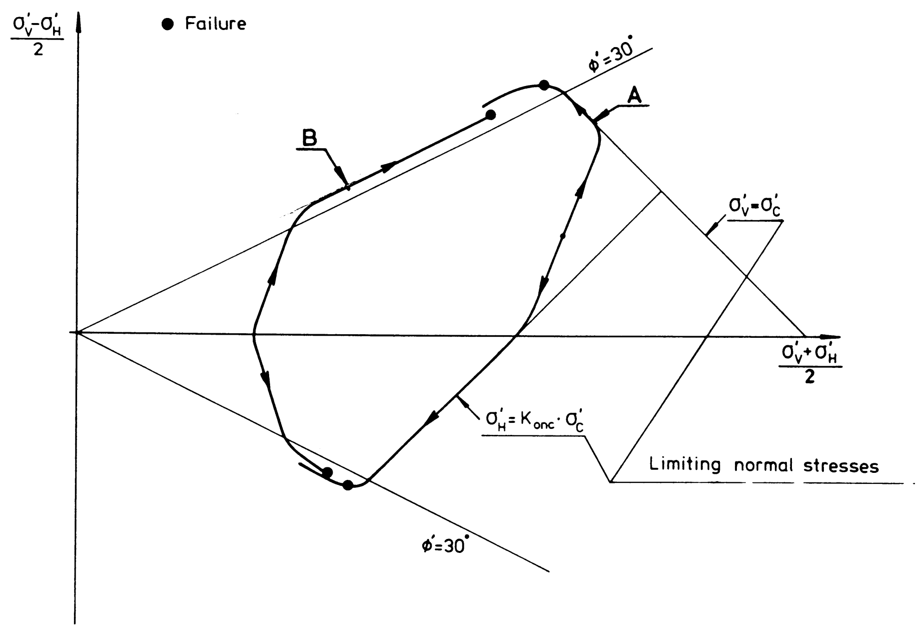


Fig. 10. Typical stress paths in soft clay in triaxial tests performed with normal rates of deformation [7].

5.3 FIELD TESTS

The development of pore pressures and deformations under a large number of embankments has been studied.

Pore pressure generation

In those cases where the soil has been slightly overconsolidated, it has been observed that the pore pressure generation has been lower than that calculated on the assumption of fully undrained conditions until the apparent preconsolidation is reached. For stresses within the elastic stress range where the compression modulus is high, a certain amount of consolidation and pore pressure dissipation can thus be assumed to take place within the normal time for load application at construction of an embankment.

For clays with overconsolidation ratios less than about 2.5, there is a radical change in the development of pore pressures when the effective vertical stress reaches the apparent preconsolidation pressure. For further stress changes, the increase in pore pressure directly corresponds to the increase in vertical stress and the effective vertical stress is thereby prevented from exceeding the apparent preconsolidation pressure, *Fig. 11.*

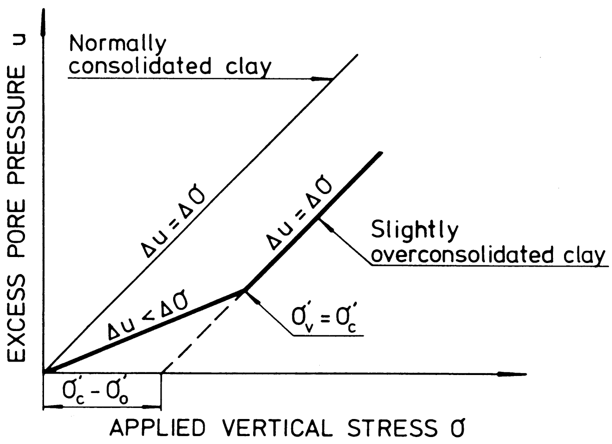


Fig. 11. Typical pore pressure development at construction of an embankment in the field [7].

The difference in maximum overconsolidation ratio for this type of pore pressure development from about 2 in the fully undrained laboratory tests to about 2.5 in the field tests depends on the partial drainage occurring in the field.

Apparent preconsolidation pressure

The observed pore pressure development in the field, as well as observed settlements for approach embankments with a very gradual increase of the load, has been used to calibrate the apparent preconsolidation pressures evaluated from the oedometer tests.

In this way, it has been possible to establish that oedometer tests performed and evaluated according to the standard procedures yield apparent preconsolidation pressures that correspond very well to the effective vertical stresses in the field where large changes occur both regarding pore pressure generation and settlements.

From measurements of the horizontal movements below the toes of the embankment slopes, it has also been possible to establish that horizontal movements corresponding to the vertical settlements will not occur until the effective vertical stresses reach the apparent preconsolidation pressures. Only then will it be relevant to assume fully undrained conditions and $\nu = 0.5$.

Deformations

Long-term observations of settlements for stress levels below the apparent preconsolidation pressure have only been performed in one case, but also these measurements indicate that the secondary compressions start to become significant at a stress level of about $0.8 \cdot \sigma'_c$ and then increase with increasing stresses, *Fig. 12*.

Long-term field observations of settlements developing at loading to stresses above the apparent preconsolidation pressure show that the settlements become considerably larger and develop much more rapidly than those calculated without consideration to time effects.

The horizontal movements have proved to continue for a long time after load application but decrease in relation to the settlement and can from a practical point of view be assumed to stop altogether after a certain period of time.

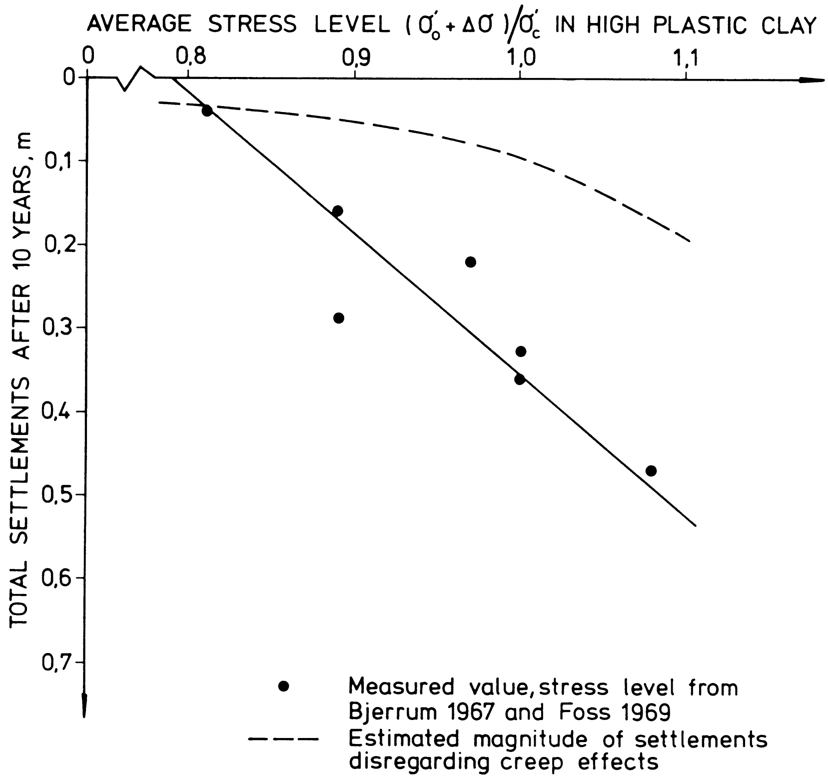


Fig. 12. 10-year settlements under buildings in Drammen as a function of the stress level in the upper high-plastic clay layer [7].

Excess pore pressure dissipation

The dissipation of excess pore pressures has been found to be a much slower process than that calculated without regard to time effects on the properties. Considerable excess pore pressures remain when the "final" settlement calculated without regard to the time effects has been reached and the settlements then continue without any tendency to slow down more than the continuous decrease in rate because the settlement is more or less linear against the logarithm of time.

Development of apparent preconsolidation pressures and undrained shear strengths

Investigations of samples taken under the embankments show that quasi-preconsolidation pressures develop which are considerably higher than the largest vertical stresses that have occurred in the soil. Also shear strength tests performed under the embankments in the field show that undrained shear strengths corresponding to these quasi-preconsolidation pressures develop.

Conclusion

In order to predict the observed development in the field, a calculation model is required which takes into account the time dependence of the soil compressibility.

Natural apparent overconsolidation

Another observation which has been made in the field is that natural soil deposits as a rule exhibit a certain overconsolidation which has been created during the time that has elapsed since the deposition by secondary consolidation for the soil´s own weight and climatic factors. This apparent overconsolidation can vary from only a few kPa in the middle of thick deposits of soft "normally consolidated" clay which have never been subjected to any extra overload and in which very slow consolidation processes continue, among other things because of the ongoing land heave. Closer to draining layers, the overconsolidation ratio OCR ($=\sigma'_c/\sigma'_0$) increases to become normally at least 1.25 at the drainage borders. For soils which have been subjected to real overloads and then been reloaded, higher overconsolidation ratios normally apply.

This overconsolidation, even when small, must be considered in calculations with time dependent parameters. Unless this is done, the calculated secondary consolidation will begin immediately at a maximum rate at the start of the calculations even for a marginal increase in the load.

5.4 INFLUENCE OF TIME DEPENDENCE OF THE COMPRESSIBILITY ON THE SIZE OF THE SETTLEMENT

Many attempts have been made to create rules of thumb for estimation of the additional settlements because of creep effects. However, there is no simple and general way of estimating the size of the settlements at different times with respect to the time dependence of the compressibility and such a way cannot be found. A soil profile cannot be simplified to a few homogeneous layers with constant properties for calculation of settlements. Even less can such a simplification be made in a comparison between settlements calculated with and without consideration to the time dependence of the parameters. This is partly due to the fact that the relations between moduli and time dependence are not constant but depend on stress level and stress history and partly to the fact that the entire consolidation process is regulated by permeability, drainage paths and the interaction between the different soil layers in the profile. The influence of the time dependence of the compressibility on the settlements and their course with time therefore becomes a unique function for each specific loading case and soil profile.

Larger settlements

Attempts to obtain rough estimates of the size of the settlements have been made [32]. As expected, the results of these attempts show that the calculated settlements become larger throughout the consolidation process when account is taken of the time dependence of the compressibility and that the relative difference increases with time. It is common for the settlement calculated with time dependent parameters to be more than twice the settlement calculated otherwise in clays with relatively high water contents. The increase in the relative difference in calculated settlements with time occurs faster when the permeability of the soil is higher. Corresponding effects can be observed in settlement observations in the field.

In general, the relative difference in the calculated settlements increases when both the moduli (M) and the time effects (α_s) increase. A higher modulus gives smaller settlements calculated without regard to the time dependence and an increase in time effects gives larger calculated additional settlements because of these effects.

Complication of the general picture by apparent overconsolidation

In normally consolidated soil, the difference between the two types of calculation generally decreases with increasing application of load. However, most soil profiles are more or less overconsolidated, particularly in layers close to the ground surface or drainage boundaries. This considerably complicates the picture. When the soil is sufficiently overconsolidated, there is no significant difference in the calculated settlements whether the time effects are observed or not. When the soil is slightly overconsolidated, the difference in calculated settlement in a specific layer reaches a maximum when the sum of the initial vertical stress and the additional stress coincides with the apparent preconsolidation pressure. For the soil profile taken as a whole, where both overconsolidation ratios and additional stresses vary, the relative difference reaches a maximum at an additional load which can vary with both the internal relations between the properties in the different layers and the considered period of time. Also factors such as the geometry of the load, the thickness of the soil layers and the stress distribution with depth, as well as the changes in load that occur during the consolidation process, play significant roles for the calculation results.

A method for estimation of the influence of the time dependence of the compressibility which is both simple and useful in practice cannot thus be created. The demand for such a method has also in practice been eliminated as a result of the development in calculation aids. Calculations which only a few years ago demanded access to a large computer can today be performed with ordinary personal computers.

5.5 EXAMPLES OF MEASURED AND CALCULATED SETTLEMENTS

Investigations and prediction of settlements are performed by the Institute both for research purposes and in consulting projects. In many cases, the settlements in the field are monitored for several years after construction, enabling a comparison between predicted and actual settlements, see for example [7], [29], [30].

In this chapter, three examples of full scale cases in the field where the settlements have been monitored are presented; one test fill which was constructed for research purposes and where the investigations are very comprehensive, and two road embankments with a more normal extent of the investigations.

<u>CASE 1:</u>

Test fill at Lilla Mellösa

This test fill with a base of 30 x 30 metres was constructed on top of 14 m of soft clay in 1947. The fill consists of 2.5 m of gravel with an original slope inclination of 1:1.5. The load increase under the crest was about 41 kPa. The settlements have then been followed for almost 50 years and the observations are continuing. Detailed investigations concerning the distribution of settlements with depth, remaining excess pore pressures and changes in the soil properties have been carried out at different times.

Supplementary investigations concerning the soil properties outside the loaded area have also been performed when new and better equipment and methods have been developed.

Soil profile

The soil profile in the natural ground at Lilla Mellösa is shown in *Fig. 13*.

On top of the profile was a 0.3 m thick layer of top soil which was scraped off before the fill was applied. The dry crust is unusually thin and consists of organic soil. The fissured and weathered part is only half a metre thick and overlies soft clay. The clay has an organic content of about 5 % immediately below the dry crust. This content decreases with depth and is less than 2 % at depths greater than 6 - 7 metres. The water content is about equal to the liquid limit and decreases from 130 % in the upper organic clay to 70 % in the bottom layers. Below 10 depth, the clay is varved. The varves are at first diffuse, but become more distinct with depth. At 14 m depth there is a thin layer of sand on top of bedrock.

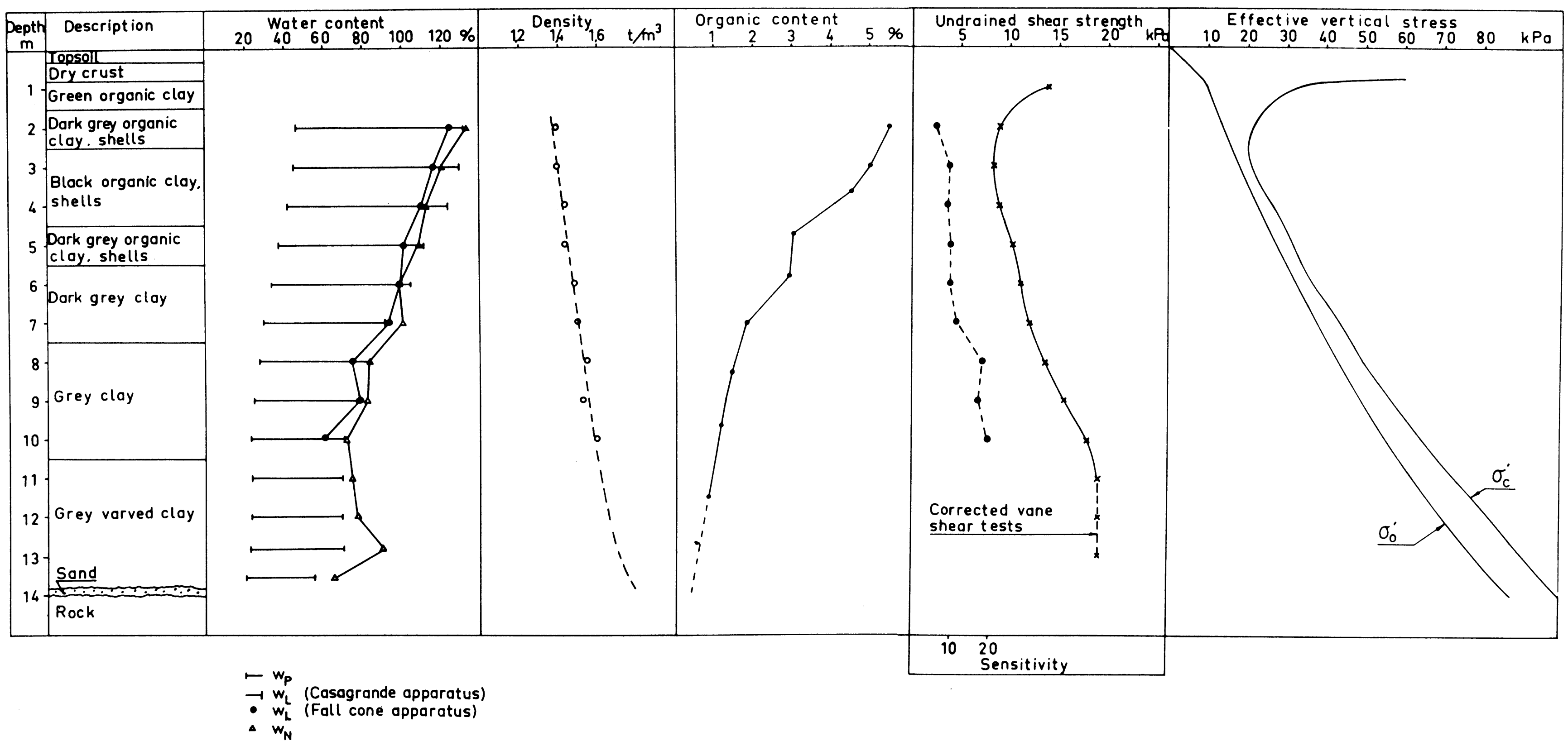


Fig. 13. Soil profile at Lilla Mellösa [7].

The undrained shear strength in the clay has a minimum of 8 kPa at 3 m depth and then increases with depth. The overconsolidation ratio in the soft clay is fairly constant at about 1.2, which entails an overconsolidation from only about 3 kPa at the top to about 12 kPa in the lower part of the profile. The pore water pressure is hydrostatic from a ground water level 0.8 m below the ground surface.

Measured and calculated settlements

The settlement during the time for load application, which lasted for 25 days, amounted to 0.07 m. The corresponding calculated settlement because of elastic deformations was 0.1 m.

Fig. 14 shows the measured and calculated settlements together with measured distributions of settlements and excess pore pressures with depth. The consolidation settlements have been calculated with and without consideration to the time dependence of the compressibility. The parameters evaluated from oedometer tests and employed in the calculations are shown in *Fig. 15*.

The "final settlement", which is calculated without regard to the time effects, is just below 1.4 m. This

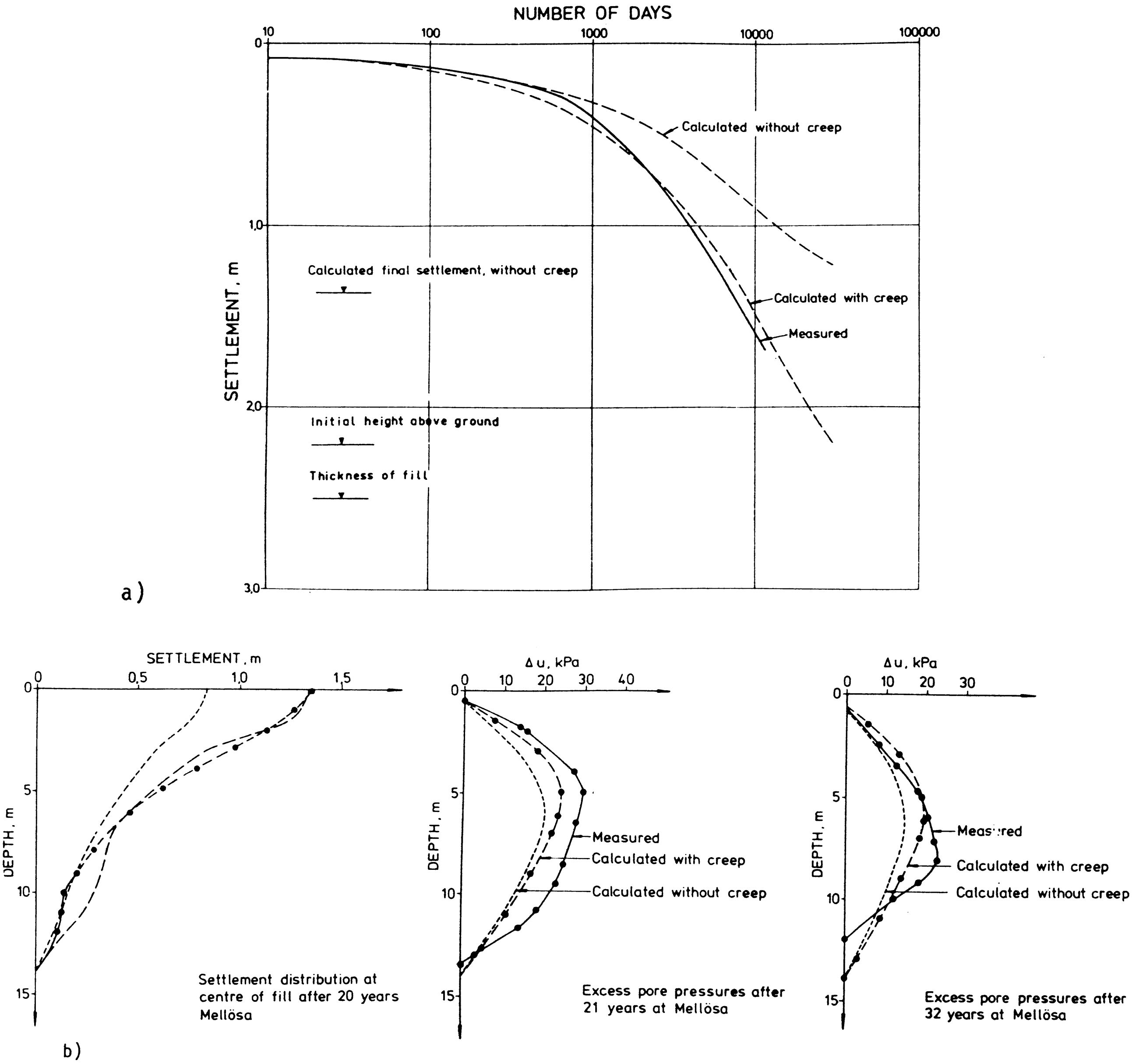


Fig. 14. Measured and calculated settlements at Lilla Mellösa [7].
a) total settlement
b) distribution of settlements and excess pore pressures in the soil profile at various times after load application.

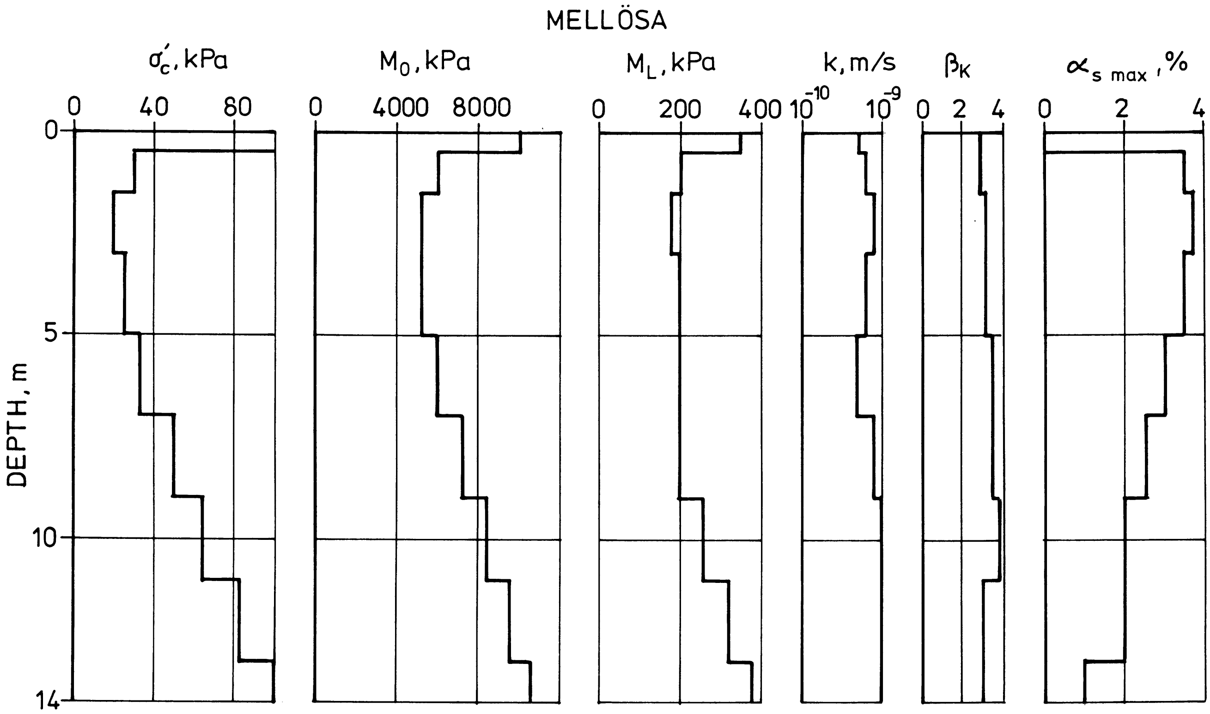


Fig. 15. Compressibility and permeability parameters at Lilla Mellösa evaluated from oedometer tests [7].

settlement was reached after 19 years (1966). At this time, the remaining excess pore pressures amounted to about 30 kPa and the settlements continued. In 1979, 32 years after the load application, the settlements amounted to 1.65 m and the remaining excess pore pressures were over 20 kPa. The settlements in 1992 amounted to 1.8 m and are still increasing.

From the results shown in Fig. 14, it is evident that the time effects on the compressibility have to be taken into consideration if a good prediction of the size of the settlements and their course with time is to be made. Otherwise, the real settlements in this case became twice as large as the calculated settlements after 10 years and onwards and with time the calculated "final settlements" were exceeded by a wide margin. The measured pore pressures show that none of this can be related to a more rapid excess pore pressure dissipation than calculated and it has been checked that it is not related to horizontal movements.

CASE 2:
Embankment on Road E4 between Råneå and Strömsund

The construction of Road E4 between Råneå and Strömsund was started in 1982. On the part of the road in question, the road embankment is 9.5 m wide and the 1.1 m high fill consists mainly of slag stone from an ironworks. The settlements are monitored in horizontal flexible tubes under the embankment. When the settlements were found to become larger than expected, the fill was adjusted within a year by an additional 0.45 m of fill. The soil profile, the results of the calculations and the measured settlements for section 4/720 - 5/180 are presented.

Soil profile

The soft soil consists of an approximately 9 m thick layer of slightly organic clay coloured by sulphide overlying coarser soil. The pore water pressure can be assumed as hydrostatic for a ground water level 0.75 m below the ground surface and the clay is almost normally consolidated from 4 m depth and below. The undrained shear strength varies between 6 and 14 kPa. The soil stratigraphy and some of the calculation parameters are shown in *Fig. 16*.

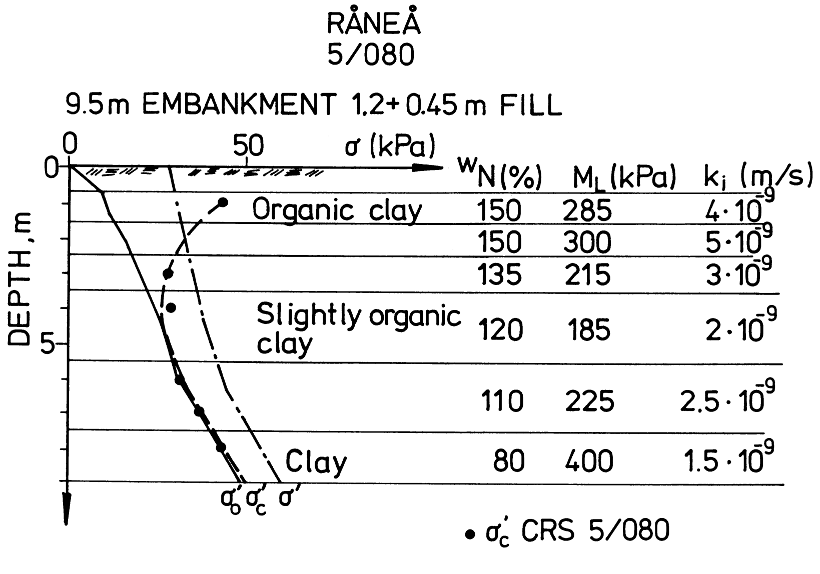


Fig. 16. Summary of the geotechnical conditions in Section 5/080 on Road E4, part Råneå-Strömsund [29].

Measured and calculated settlements

Fig. 17 shows the average of the measured settlements in the centre line of the road together with calculated settlements with and without consideration to time effects on the compressibility. As is evident from the figure, the actual settlement is much larger and occurs much more rapidly than that calculated without regard to the time dependence of the parameters. After about 5 years, the measured settlements were more than twice the settlements calculated in such a way, 0.75 m versus 0.33 m respectively, and the difference increases with time. Calculations taking the time dependence of the compressibility into account yield settlements and courses with time that are in acceptable agreement with the measurements.

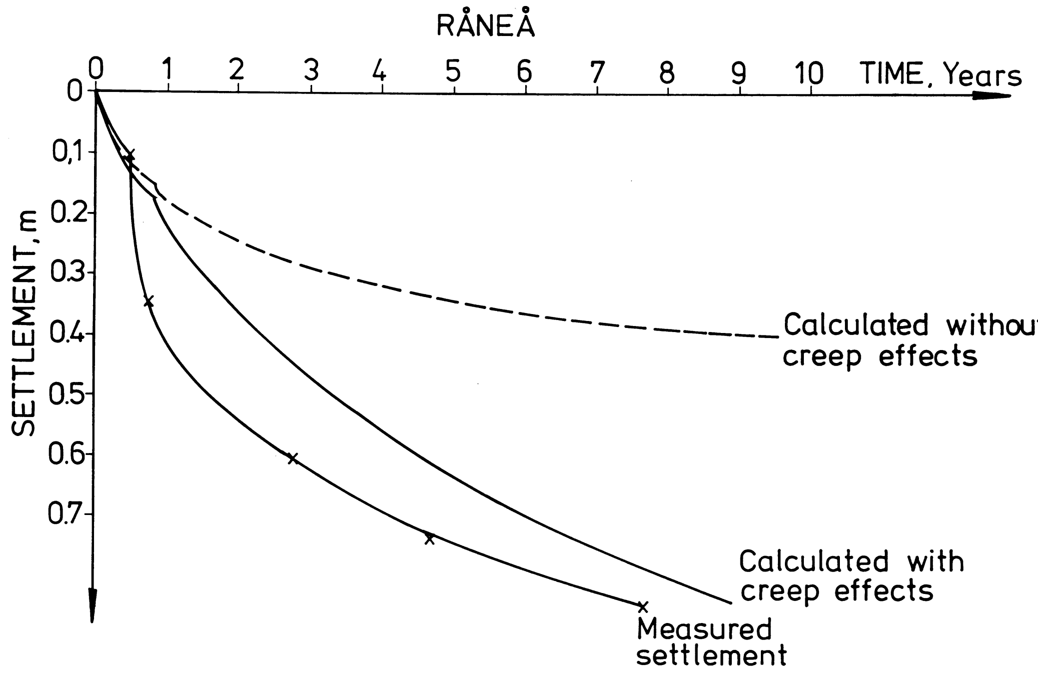


Figure 17. Time-settlement curves for part 4/750-5/180 on Road E4, part Råneå-Strömsund [29].

CASE 3:

Embankment on Road E3 between Västra Åby and Adolfsberg

The embankment is on a stretch of motorway between Mariestad and Örebro. Road E3 has here dual embankments, each with a crest width of 12 m. The central dividing strip has a width of 16 m and the height of the embankments is 1.4 m. The section in question, 20/800, was constructed without any soil improvement but is located relatively close to a section where vertical drains and surcharging have been applied. The sequence of load application during construction is not known in detail but the first application of fill started in July 1979 and the road was opened for traffic 2 years later. The settlements are being followed by measurements in horizontal flexible tubes under the embankments.

Soil profile

In the particular section, a 5.5 to 7 m thick clay layer lies on top of coarser soil. A firm dry crust with a thickness of about 1m has developed. The pore water pressure is hydrostatic from 1 m below the ground surface and the clay is slightly overconsolidated with apparent preconsolidation pressures about 10 kPa higher than the in situ vertical stress from 2 m depth and below. The undrained shear strength varies between 10 and 18 kPa. The stratigraphy and some of the calculation parameters are shown in *Fig. 18.*

Measured and calculated settlements

The measured and calculated settlements in the centre line of the embankment are shown in *Fig. 19*. The settlements calculated on the assumption of a smooth and even load application correspond poorly to what was measured in the field during the time for construction and directly thereafter. However, the long term observations clearly show that the time dependence of the compressibility has to be taken into account if a good correlation between predicted and actual settlements is to be obtained.

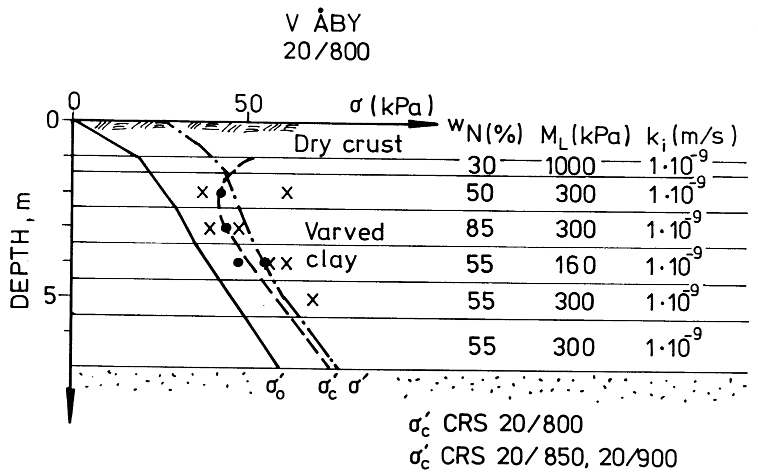


Fig. 18. Summary of the geotechnical conditions in Section 20/800 of Road E3, part V. Åby-Adolfsberg [30].

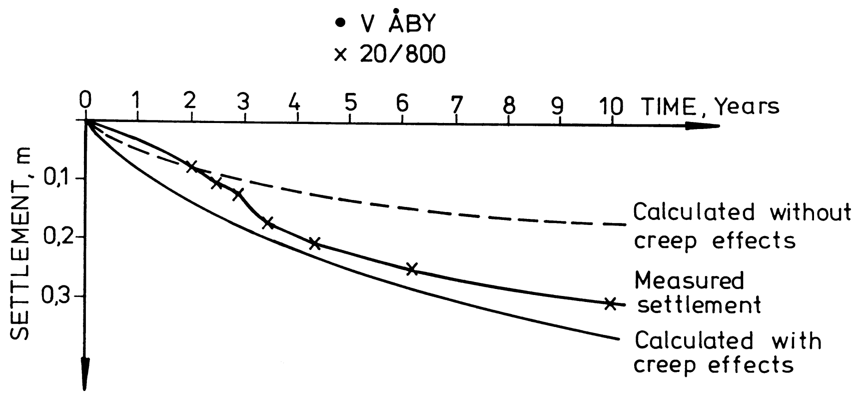


Fig. 19. Course of settlements with time in Section 20/800 on Road E3, part V. Åby-Adolfsberg [30]

CHAPTER 6.
Calculation of stress distribution at loading

A careful calculation of the distribution of additional stresses in different parts of the soil mass below an embankment is of very great importance for predictions of both the maximum size and the rate of the settlements and also for the distribution of settlements across the embankment.

These calculations are made on the basis of the theory of elasticity which, in spite of the fact that a soil mass does not directly correspond to the assumptions of an isotropic, ideal elastic and weightless material, is the most developed method available at present.

Calculation of vertical stresses under embankments with arbitrary cross sections is performed by integration of the Boussinesque solution for stresses caused by strip loads on a semi-infinite, homogeneous and isotropic mass [19], *Fig. 20*.

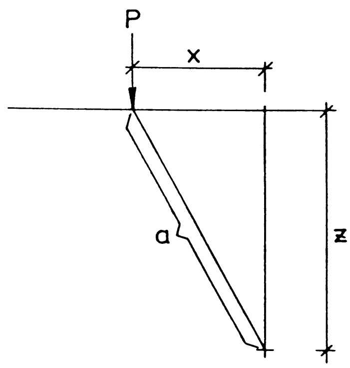


Fig. 20. Designations in the Boussinesque solution for stresses caused by strip loads on a semi-infinite homogeneous isotropic mass.

ESTIMATION OF STRESS DISTRIBUTION BY USING CHARTS

In calculations by hand, it is possible to use one of the charts which have been constructed for this purpose, e.g. the Osterberg chart [20], *Figs. 21* and *22*.

The Osterberg chart is used in such a way that the influences of several loads with the given shape are considered and the influence factors are summated.

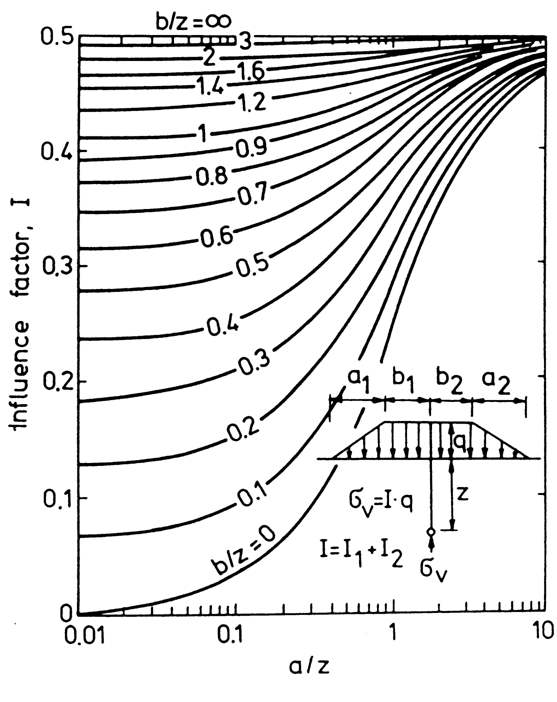
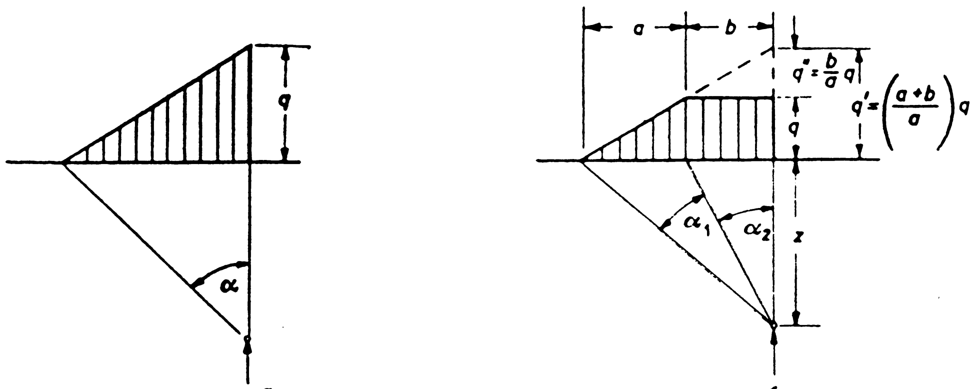


Fig. 21. The Osterberg influence chart for estimation of stresses under embankments. The chart is constructed by using the solution for the influence of a triangular load of infinite length [20].



$$\sigma_z = \frac{q}{\pi}\alpha$$

$$\sigma_z = I \cdot q$$

$$\sigma_z = \frac{q}{\pi}\left[\left(\frac{a+b}{a}\right)(\alpha_1 + \alpha_2) - \frac{b}{a}\alpha_2\right]$$

$$\therefore I = \frac{1}{\pi}\left[\left(\frac{a+b}{a}\right)(\alpha_1 + \alpha_2) - \frac{b}{a}\alpha_2\right]$$

$$I = f\left(\frac{a}{z}, \frac{b}{z}\right)$$

Fig. 22. Equations used in the construction of the Osterberg chart [20].

EXAMPLES
of calculations of stress distribution

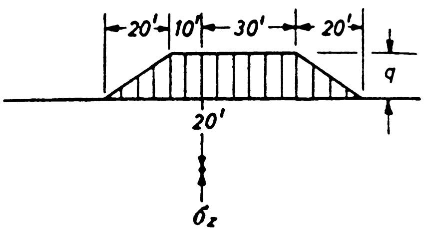


Fig. A

Example A: Calculate the additional vertical stress at the point under an embankment shown in Fig. A. At the left hand side, $a/z = 1$, $b/z = 0.5$ and the corresponding $I = 0.397$ is obtained from the chart. For the right hand side, I becomes 0.478 and the total influence factor is 0.397 + 0.478 = 0.875. The additional vertical stress is $s_z = 0.875 \cdot q$.

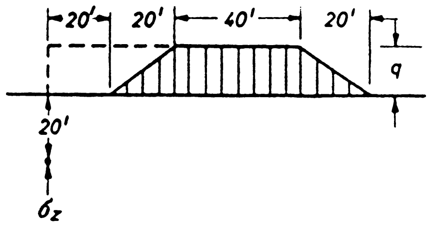


Fig. B

Example B: Calculate the additional vertical stress at the point outside the loaded area shown in Fig. B. First determine the influence factor for the embankment together with the dashed imaginary part ($a/z = 1$, $b/z = 4$, $I = 0.499$) and then deduct the influence value from the dashed imaginary part ($a/z = 1$, $b/z = 1$, $I = 0.455$). The additional vertical stress is $s_z = 0.044 \cdot q$.

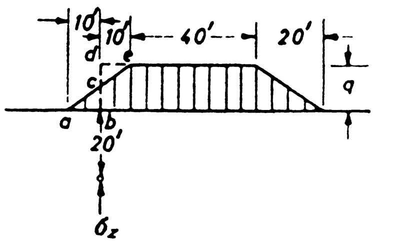


Fig. C

Example C: Calculate the additional vertical stress at a point under the centre of the slope shown in Fig. C. The stress caused by the triangle *abc* is equal to the stress caused by the imaginary triangle *cde*. Hence, the additional stress can be calculated from the influence factor from a load with $a/z = 1$ and $b/z = 2.5$. The additional vertical stress is $s_z = 0.492 \cdot q$.

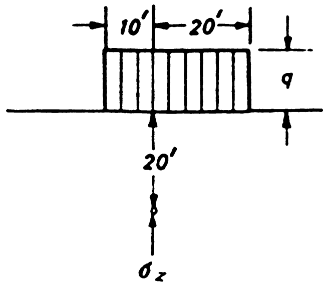


Fig. D

Example D: The chart can also be used for evenly distributed loads. The additional stress at the point shown in Fig. D is calculated using the limit value for a/z = 0.01. The error because the real value of a/z is 0 becomes a 4 % too high influence factor when b/z = 0.1, 2 % too high when b/z = 0.3 and 1 % when b/z = 0.5. For higher values of b/z, the error becomes negligible. The influence factors in the example become b/z = 0.5, I = 0.278·0.99 = 0.275 and b/z = 1.0, I = 0.410·1.00 respectively. The additional vertical stress becomes $s_z = 0.685 \cdot q$.

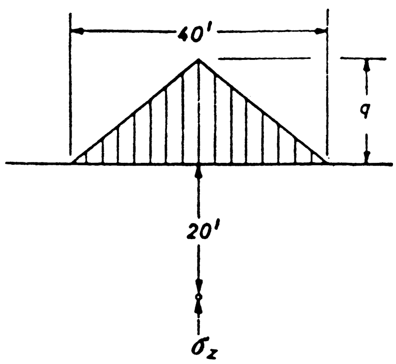


Fig. E

Example E: $b/z = 0$, $a/z = 1$, $I_1 = I_2 = 0.25$. $s_z = 0.5 \cdot q$.

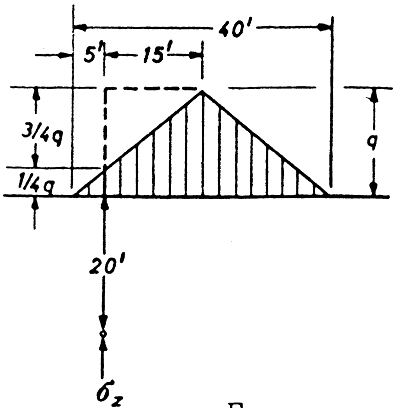


Fig. F

Example F: The load is divided into two parts and an imaginary part is added. $I = 0.08 \cdot 1/4 + 0.474 - 0.203 \cdot 3/4$. $s_z = 0.302 \cdot q$.

Influence of dry crust

The calculations according to the theory of elasticity often have to be modified with consideration to the fact that the soil in the profile often cannot be simplified to a semi-infinite medium with ideal elasticity. When soft clays are covered by a firm dry crust, this affects the stress distribution. The type and extent of this effect depend on the character of the crust. In a strongly fissured crust, no spreading of the load occurs and in calculation of the stress distribution at greater depths, the load can be assumed to be applied at the level where the wide open cracks disappear.

The depth to this level is normally very limited. For estimation of the influence of a more homogeneous dry crust on soft soil, an analogy can be made in which the crust is assumed to act as a beam with a certain stiffness against deflection. The dry crust can then be replaced by a layer with the same modulus of elasticity as the underlying soil, but with the fictitious height h_e

$$h_e = 0.9h(E1 / E2)^{1/3} \qquad [21]$$

where 0.9 = empirical correction factor
h = actual thickness of dry crust
E_1 = modulus of elasticity in dry crust
E_2 = modulus of elasticity in underlying soil

The calculation of stress distribution is then performed as for an ideally elastic soil but with the depth in the dry crust corrected according to $z = z_1 \cdot h_e / h$ and in the underlying soil corrected according to $z = z_1 + h_e - h$.

The soil can be divided into several layers according to the same principle, provided that the stiffness decreases with depth.

Limited depth to firm bottom

When the depth to firm bottom is limited, also this circumstance affects the spreading of the load effects and the stress distribution. When there is a firm bottom at a limited depth (h_f) below the compressible soil, the additional stresses can be calculated by a correction of the depth in the calculations according to the theory of elasticity for a semi-infinite medium. The influence of the stiff base extends upward to about half the depth, ($0.5 \cdot h_f$). From this level, the additional stress is calculated both for depth z and for the corrected depth $0.75 \cdot z$. The additional stress at depth z in the lower half of the compressible layer is then calculated by interpolation between these values according to

$$\sigma = \sigma_z + (\sigma_{0.75z} - \sigma_z)\frac{z - 0.5h_f}{0.5h_f}$$

Also in cases where the stiffness of the soil increases rapidly with depth, similar but more limited stress concentrations may occur. The additional stresses can then be calculated in accordance with the type of generalized Boussinesque solution presented by Fröhlich [22] among others.

Simplified methods

The type of simplified approximate methods that have been presented, such as the 2:1 method, cannot be used to calculate the stress distribution across the section and should also be avoided in calculations of settlements on the whole. Because the original in situ vertical stress, the overconsolidation ratio, the apparent preconsolidation pressure and the compressibility all vary with depth, a relatively small change in calculated additional stress can result in large differences in calculated settlement.

Accurate determination of the existing in situ vertical stresses and the pore pressures is as important as correct calculation of the additional stresses.

Submersion below the ground water level

When the settlements cause a depression of the upper soil layers and/or parts of the embankment below a high free ground water level, the load intensity q is reduced by

$$\Delta q = s \cdot (\gamma - \gamma_m + \gamma_w)$$

where
- q = load intensity, kPa
- s = total settlement, m
- γ = unit weight, kN/m^3
- γ_m = unit weight of water saturated material, kN/m^3
- γ_w = unit weight of water, kN/m^3

> The EMBANKCO programme uses the theory of elasticity and the Boussinesque solution for stresses caused by strip loads on a semi-infinite homogeneous isotropic mass. Corrections are applied for limited depth to firm bottom and for submersion below the ground water table.

Chapter 7.
Calculation of initial deformations

At loading of a limited area of the ground, shear deformations will occur. The settlements in an embankment resulting from so-called elastic shear deformations, where horizontal movements occur under the toes of the slopes and corresponding vertical movements occur in the embankment, are calculated by using the theory of elasticity. It is normally assumed that these deformations occur in a fully undrained mode, $\nu = 0.5$, and in spite of the fact that these deformations are partly time-dependent, they are calculated as if they occurred at the same rate as the load is applied.

The elastic settlements are usually small when normal safety against shear failure is applied but may be considerable at low factors of safety and in organic and high-plastic soil. In calculation of the initial settlements, the loaded area is divided into rectangular parts, each with one corner at the point for which the settlement is to be calculated [23], *Fig. 23*.

On the assumption that $\nu = 0.5$, the settlement at the corner point is then calculated for the four rectangles as

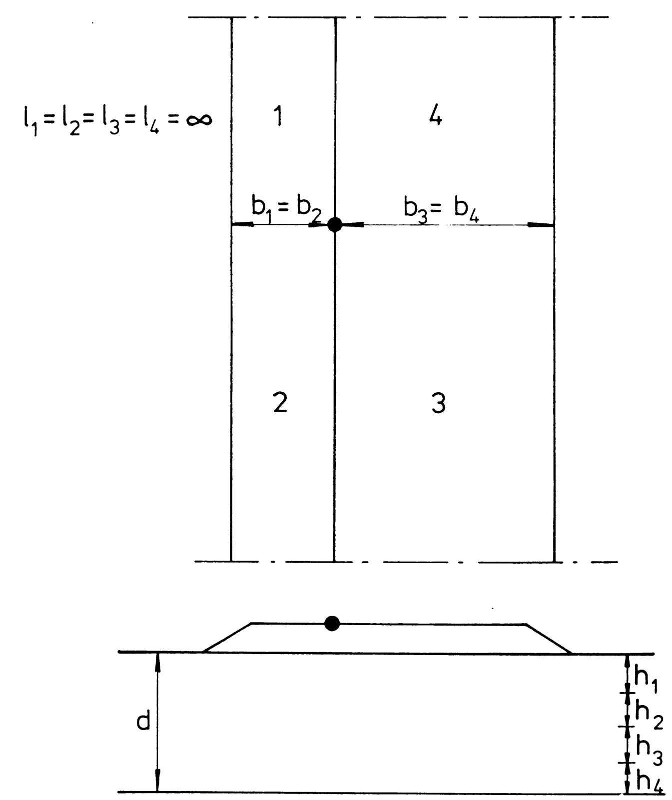


Fig. 23. Division of an embankment into rectangular load areas.

$$S_c = \frac{q \cdot b}{E} \cdot 0.75 \cdot F_1$$

The factor F_1 for $l/b = \infty$ is obtained from *Fig. 24.*

The calculated settlements for the common corner in the four rectangles are then summed up in order to obtain the settlement at this particular point.

The division into rectangles can be further elaborated in cases where the geometry and the load intensity vary.

The curves shown in *Fig. 24* are only valid for long embankments and loads with similar geometry. In principle, the method can be used for all types of geometry and ν-values, but with different functions for the influence factors. These can be studied in, for example, SGI Meddelande No. 10 from 1972 and the handbook BYGG from the same year [24].

It has to be taken into consideration that the modulus of elasticity varies between different layers. At a limited depth ($d/b \leq 3$), the influence factor increases approximately linearly with depth and a characteristic modulus of elasticity can then be chosen as a harmonic mean value over the entire depth.

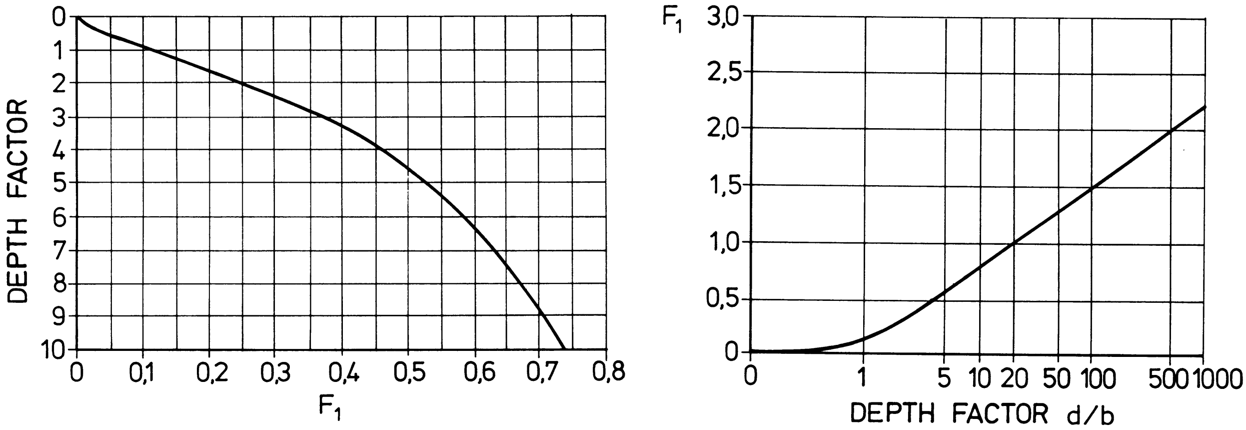


Fig. 24. Chart for estimation of factor F_l when l/b = ∞.

$$\frac{d}{E} = \frac{h_1}{E_1} + \frac{h_2}{E_2} + \frac{h_3}{E_3} + \ldots\ldots\ldots + \frac{h_n}{E_n}$$

At greater relative depths and in more careful calculations, the settlement is calculated by a summating the settlement in the various layers as

$$S_c = q \cdot b \cdot 0.75\left(\sum_{i=1}^{n-1} \frac{(F_{i+1} - F_i)}{E_i} + \frac{F_{i_n}}{E_n}\right)$$

where E_i = modulus of elasticity in layer i
F_i = influence factor at depth d_i, which is equal to the distance from the ground surface to the upper edge of layer i.

Calculation of initial deformations because of horizontal movements is normally not included in calculation programmes for one-dimensional consolidation; instead, these settlements must be calculated separately. This is the case also in the EMBANKCO programme. In more careful predictions and where the size of the initial settlement is of importance, the initial deformations are calculated beforehand. The subsequent calculation of the consolidation process is then performed with a load and a geometry which have been adjusted with consideration to the initial settlements.

CHAPTER 8.

Calculation of the course of settlements in one-dimensional consoladition

8.1 PRINCIPLE OF CALCULATION OF THE COURSE OF THE CONSOLIDATION PROCESS

The calculation of the consolidation process for embankments on soft soils is based on the fact that the pore water pressure increases because of the applied load and the resulting pressure gradient gives rise to a flow of water out of the soil. This flow of water is normally assumed to take place only in the vertical direction. When draining layers are embedded in the soil profile, the water flow is assumed to be vertical between these layers.

In cases where the loaded area is small in relation to the thickness of the compressible layers, the influence of the horizontal water flow cannot be neglected. This is particularly the case for circular and square footings. In the case of embankments, the horizontal water flow apart from the flow in the draining layers can normally be neglected. However, there is always a certain influence from the horizontal water flow on the rate of the settlements in the outer parts of the embankment. The settlements in these parts therefore become somewhat larger and occur somewhat more rapidly than those calculated on the assumption of one-dimensional consolidation.

Calculation for a single layer with constant parameters

In calculation of the consolidation process without regard to time effects on the compressibility the following equation is used [1] :

$$\frac{\partial u}{\partial t} = -M \frac{\partial v}{\partial z}$$

or

$$\frac{\partial u}{\partial t} = -\frac{M}{g \cdot \rho_w} \cdot \frac{\partial}{\partial z}(k \frac{\partial u}{\partial z})$$

where u = excess pore pressure
v = velocity of pore water flow
t = time
k = permeability
M = oedometer modulus
ρ_w = density of pore water
z = distance from the draining surface to the particular element (*Fig. 25*).

The calculation result in terms of degree of dissipation of the excess pore pressure after a certain time (degree of consolidation) becomes a function of the initial distribution of excess pore pressure with depth, the drainage paths and the compressibility and permeability of the soil. The time factor T_v is calculated from

$$T_v = \frac{k \cdot M}{g \cdot \rho_w} \cdot \frac{t}{d^2}$$

see *Fig. 26*.

Calculation for multi-layer systems

In most cases, the soil in a profile cannot be simplified to a single layer with constant modulus and permeability, but has to be divided into several layers with different properties. It is then no longer possible to solve the equation analytically and a numerical procedure has to be applied. A method for this purpose, by which the consolidation process can be determined through a graphical procedure, was presented by Helenelund in 1951, [27].

In this method, the soil profile is divided into a number of layers in such a way that the product

$$\frac{k \cdot M}{g \cdot \rho_w}$$

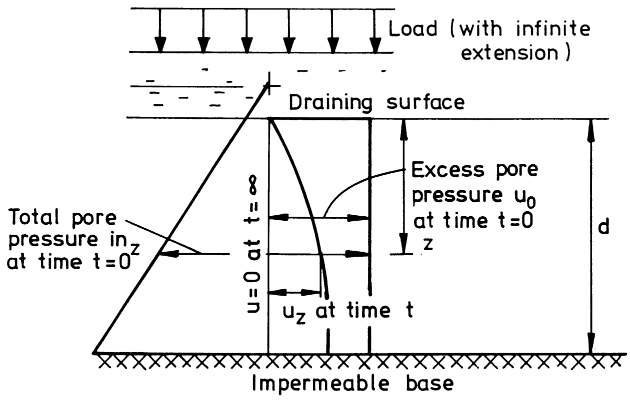


Fig. 25. Distribution of excess pore pressure in a clay layer with single drainage at the top caused by an applied surface load with a large extent in relation to the thickness of the compressible layer [25].

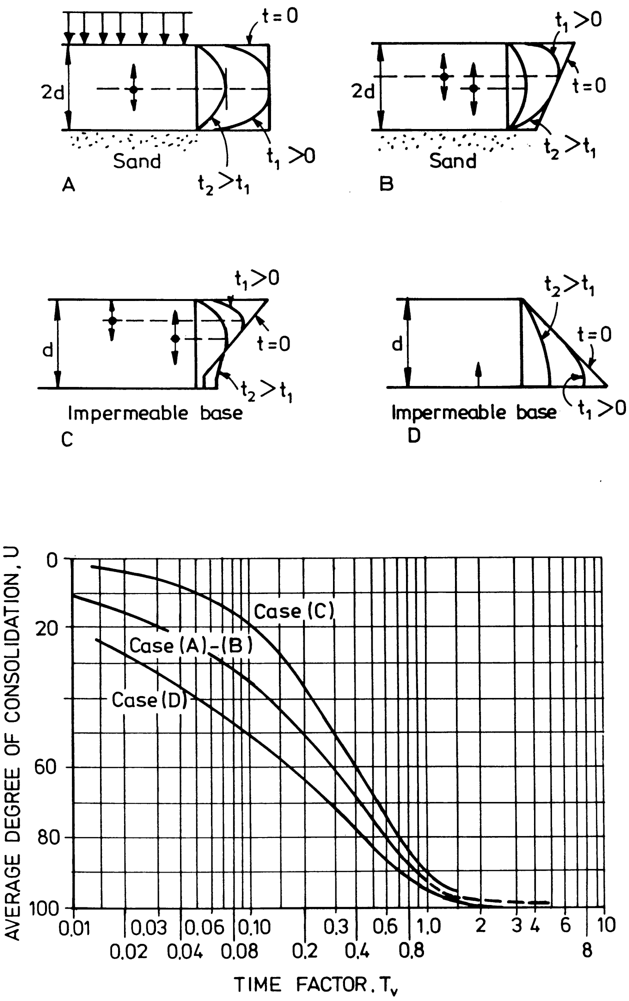


Fig. 26. Time factor based on the pore pressure equation for one-dimensional consolidation according to Terzaghi [25].

can be assumed to be constant. By choosing a constant value for ΔT_v and calculating as if all layers have a thickness of Δz, it is possible to calculate the remaining excess pore pressure after a certain time starting from the excess pore pressure at time $t = 0$. The pore pressure at the centre of the layer after time $t + \Delta t$ is obtained as the arithmetic mean of the pore pressures at the interfaces towards neighbouring layers. Because of the constant value of T_v, the thickness of the layers varies with consideration to oedometer modulus and permeability in such a way that

$$\Delta z_n = F(k_n \cdot M_n)^{0.5}$$

where Δz_n = thickness of the layer
F = a factor which is common for all layers
k_n = permeability in the layer
M_n = oedometer modulus in the layer

When the permeability in the different layers varies, the pore pressure gradients at the interfaces have to be modified. The pore water flow is equal on both sides of the interface and according to Darcy´s law

$$k_n(\frac{\partial u}{\partial z})_n = k_{n+1}(\frac{\partial u}{\partial z})_{n+1}$$

A graphical construction of this kind is shown in *Fig. 27.*

In the figure, the excess pore pressure at time $t = 0$ has been drawn as a function of depth z. Layers in which the product of k and M can be considered constant have been selected and the thicknesses of the layers (Δz) have been selected with consideration to the value of the square root of $(k{\cdot}M)$ in relation to the corresponding value for the other layers.

Auxiliary construction lines have been drawn at the centres of the layers and at distances of $(k_{n+1} / k_n) \cdot (z_n / 2)$ above the interfaces.

The excess pore pressures at the interfaces are calculated for the time step Δt and the average of the pressures at the interfaces for each layer is plotted at the centre of the layer. The excess pore pressure profile at the interfaces is constructed by using the auxiliary construction lines, whereby

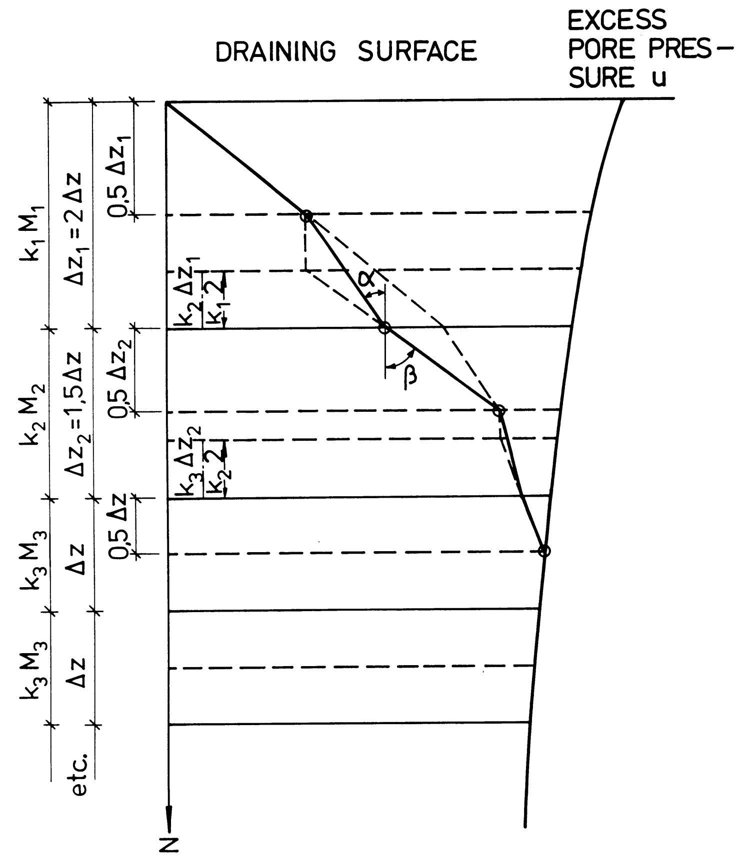


Fig. 27. The Helenelund graphical method for estimation of the consolidation process [27].

applied loads or load reductions. Such changes in applied load bring changes in the excess pore pressures.

When the compressions occur at such slow rates and at such stresses that creep effects occur, also the resulting changes in compressibility have to be considered. However, also time dependent creep compressions require that a corresponding amount of water flows out of the soil and therefore these effects in the short time perspective instead result in a corresponding increase in the excess pore pressure. Even if it is possible to calculate the consolidation process for a short step in time with constant parameters and boundary conditions, it therefore becomes necessary to update the parameters and conditions with regard to what has happened during the elapsed time and to reformat the problem before a new calculation can be made for the next step in time.

The load from an embankment is not applied momentarily but gradually during a certain time for construction. This can be taken into account by using very short time steps during this phase and updating the pore pressures with regard to the applied load during each time step. A graphical construction, which has previously often been applied for correction of the consolidation process calculated on the assumption of an instantaneous load application (broken line c_1), is shown in *Fig. 28*.

$\tan \alpha = (\delta u/\delta z)_n$ and $\tan \beta = (\delta u/\delta z)_{n+1}$. This construction ensures that $k_n \cdot \tan \alpha = k_{n+1} \cdot \tan \beta$ and thereby Darcy´s law is followed.

Thereafter, the decrease in pore pressure at the centre each layer can be measured. This corresponds to the increase in effective stress and the compression of the layer can be calculated by the oedometer modulus *M*.

Calculation with consideration to varying parameters

A compression entails that both the properties of the soil and the boundary conditions change. The compression brings a decrease in permeability and a change in the oedometer modulus which depends on the stress level. The thickness of the soil layer and the length of the drainage paths decrease. In addition, the applied load may change during the course of consolidation. This may be the result of an indirect stress reduction because parts of the upper soil layers and / or the fill material have been submerged under a high ground water level or a direct change because of

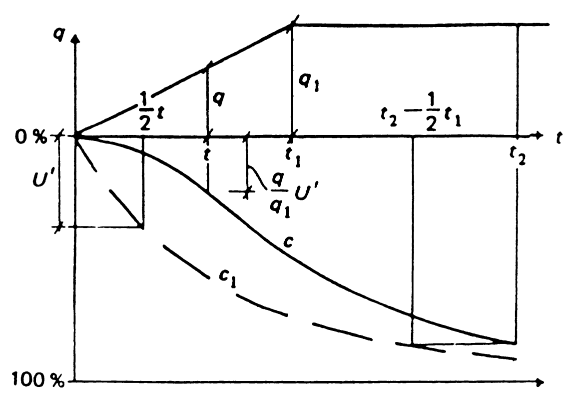


Fig. 28. Graphical construction of the consolidation curve in cases where the load is applied slowly according to Terzaghi [26].

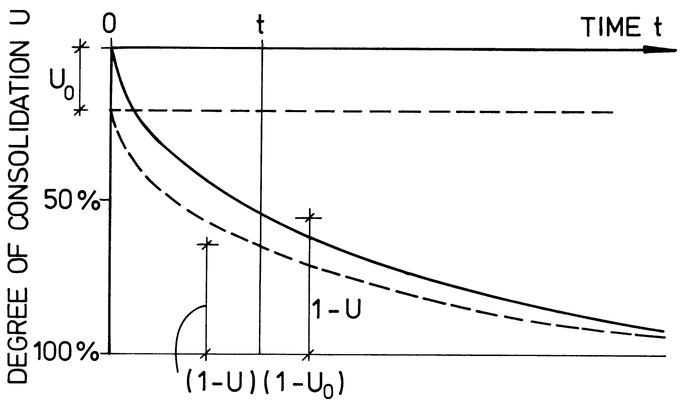


Fig. 29. Graphical construction of the consolidation curve when the soil contains gas according to Terzaghi [26]. U_0 = apparent consolidation at time t = 0 because of the compression of the gas bubbles.

When the soil contains significant amounts of gas, the consolidation process changes. This, however, is rather unusual in Swedish clay and gyttja. When the ground surface is loaded, the increasing pore pressure results in an immediate compression of the gas bubbles according to Boyle´s law and simultaneously a corresponding immediate compression of the soil mass occurs. As the excess pore pressure dissipates, the gas volume then increases, which entails that the outflow of water thereafter occurs without a fully corresponding settlement and excess pore pressure dissipation. Furthermore, the permeability changes with the size of the gas bubbles, but this is more difficult to take into consideration. The corresponding graphical construction for correction of the calculated consolidation process is shown in *Fig. 29*.

8.2 MODERN CALCULATION METHODS

The accuracy with which the courses of the settlements can be calculated depends on how well the soil and the boundary conditions can be described, how well the stress distribution can be calculated and how well the changes in properties and boundary conditions that occur during this course can be taken into account.

The possibility of performing detailed calculations by manual or graphical methods is for practical reasons very limited. With access to modern aids in terms of computers and data processing technique this limitation has virtually ceased to exist.

Advanced computer programmes for calculation of the one-dimensional consolidation process in multi-layer systems were developed in 1978 [28]. By using these programmes, it has become possible to divide the soil profile into a large number of layers with varying properties and degrees of saturation. Loads can be applied gradually in different loading steps and with varying geometry and the consolidation processes can be calculated in many small steps in time, after which the properties and the boundary conditions are continuously updated. Compared to previous calculation methods with simplified assumptions, the results from the new type of calculations have in all respects proved to correspond considerably better to the observations that have been made in full scale applications in the field concerning courses of settlements and pore pressure dissipation [7, 29, 30]. The new calculation method has also been used as a standard procedure for several years in calculations of courses of settlements for road embankments in Sweden.

The Embankco calculation programme

With consideration to the experience gained and particularly to the demand for a more user-friendly programme that can be used in the personal computers of today, the Embankco programme has been developed in co-operation between the Swedish National Road Administration and SGI. The programme is based on the theories and empirical experience that have been presented in this publication.

Conditions for the calculations

In the calculations, the somewhat simplified soil model described in Section 5.2, page 23, is used. The simplification mainly concerns the description of the course of swelling at unloading.

The **compressibility** of the soil is described by the parameters M_0, σ'_c, M_L, σ'_L, and M', as evaluated from oedometer tests according to Swedish standard. For soft normally consolidated and only slightly overconsolidated soils, the parameter M_0 may be estimated by empirical correlations. The load factor b is normally assumed to be constant = 0.8.

The compressibility of layers of coarser soil may be described by the compression modulus which is evaluated from results from in-situ methods, primarily dilatometer tests [13]. In these layers, M_0 is given as the evaluated compression modulus and the apparent preconsolidation pressure σ'_c is set to $\sigma'_c > (\sigma'_0+\Delta\sigma)$. If no tests have been performed in the coarser soil, a rough estimate of the modulus number m can be made from empirical relations. M_0 is then estimated from

$$M_0 \approx m \cdot 100\left(\frac{\sigma'_0+\Delta\sigma/2}{100}\right)^{1-\beta}$$

where m and β are obtained from *Fig. 30* [31]. σ'_c is given in the same way as $> (\sigma'_0+\Delta\sigma)$.

The **permeability** of the soil is described by its initial permeability k_i and the index for change in permeability with compression, β_k; both parameters normally being evaluated from oedometer tests.

The **creep properties** of the soil (time dependence of the compressibility) are described by the parameters $\alpha_{s(max)}$ and $\beta_{\alpha s}$. These parameters are normally estimated from empirical relations based on soil type and natural water content.

The distribution of additional stresses in the soil caused by the applied load is calculated according to theory of elasticity and the instantaneous increase in pore pressure at load application is calculated according to

$$\Delta u = \Delta p \quad \text{when } \sigma'_v < \sigma'_c$$
$$\text{and} \quad \Delta u = \Delta\sigma_v \quad \text{when } \sigma'_v = \sigma'_c$$

In analogy with the observed pore pressure development, this entails that in undrained conditions the effective vertical stress can reach but not exceed the apparent preconsolidation pressure.

(In non-saturated soil, the initial settlement because of the compression of the gas bubbles is calculated and the pore water pressures are modified accordingly.)*

The **time dependent consolidation process** is calculated in analogy with the empirical models for the time dependence of the compressibility. The applied oedometer moduli include all the time effects occuring at compressions at rates higher than $\alpha_s \cdot 5 \cdot 10^{-6}$ 1/s. This reference rate is compared to the calculated rates of compression and when the latter become lower than the reference rate, time dependent compressions are assumed to be added.

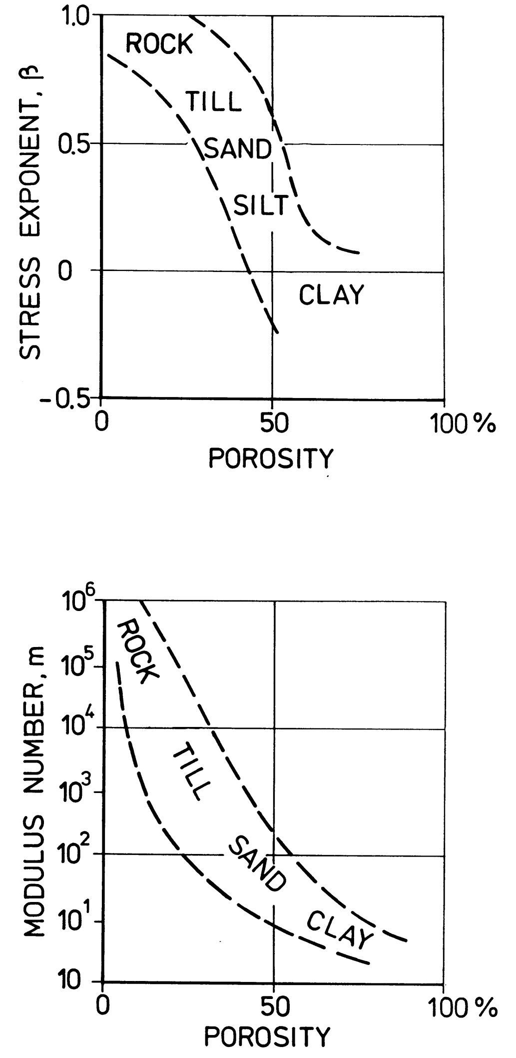


Fig. 30. Estimation of parameters *m* and from soil type and porosity according to Janbu [31].

At compression, the time dependence leads to a larger compression than that calculated from the compression moduli alone. A condition for this extra compression to occur is that the corresponding amount of water flows out of the soil. In turn, a condition for this to occur within the same period of time as the compression corresponding to the moduli alone is that a higher gradient exists in the pore water. The immediate effect of the creep tendency (or time effects) during a short time step is therefore an increase in pore pressure, whose size is determined by the potential creep deformation and the compression modulus, *Fig. 31*.

*) *Operations within parenthesis are not included in the* EMBANKCO *programme version 1. In this programme, the soil is assumed to be fully saturated. Elastic deformations are calculated separately when required.*

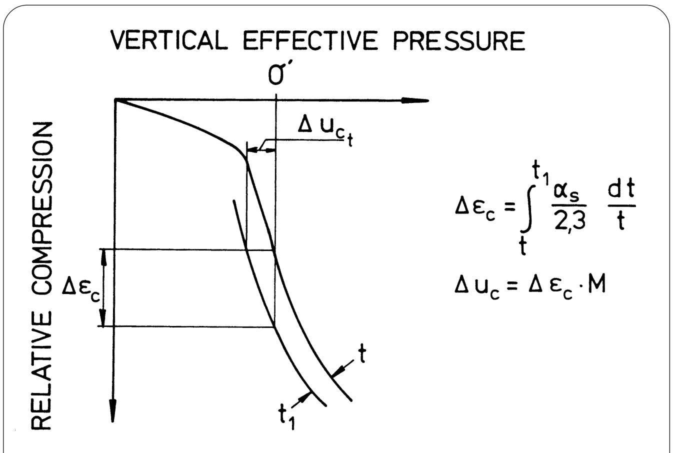


Fig. 31. Influence of the time dependence of the compressibility during a short time step [7].

The consolidation equation then becomes

where u_{ct} is the increase in pore pressure because of the creep effects. (When the soil is not fully saturated, this equation for excess pore pressure dissipation is further modified with consideration to the volume change of the gas bubbles at changes in the pore pressure.)*

As an example, a 1 m thick sub-layer can be studied. The dissipation of excess pore pressure during a time step is 2 kPa. Assuming that the oedometer modulus *M* is 500 kPa, the compression of the sub-layer during the time step is 2·1000/500 = 4 mm. During this time step, there would also have occurred a creep deformation $\Delta\varepsilon_c$ of 0.003 (= 0.3% or 3 mm) without consideration to the demand for a corresponding outflow of pore water. Instead, the pore pressure during the time step will become an increase of Δu_{ct} = 0,003·500 =1.5 kPa because of this creep effect. During the particular time step, there has thus occurred a compression of the sub-layer of 4 mm although the excess pore pressure has decreased only by 0.5 kPa.

*) *Operations within parenthesis are not included in the* Embankco *programme version 1. In this programme, the soil is assumed to be fully saturated. Elastic deformations are calculated separately when required.*

In the next time step, the pore pressure will be 1.5 kPa higher than if no creep effects had occurred in the previous time step. The rate of water flow and thereby the rate of compression of the soil will be higher because of the larger gradient and the settlement that developes will be correspondingly larger. If further creep effects occur during this time step, the excess pore pressure dissipation will be correspondingly lower, which in turn will affect the rate of settlement in the subsequent time step, and so on.

The courses of consolidation calculated with and without consideration to creep effects are shown schematically in *Fig. 32* and the calculated stress-strain relations followed during these courses are compared to the oedometer curve in *Fig. 33*.

The time dependence of the compressibility may entail that the pore pressure during a time step tends to increase, in spite of the fact that no extra stress is applied. In this case, the effective vertical stress decreases, whereby the creep process decreases and after a moderate decrease in effective vertical stress it stops altogether. When the pore pressure again starts to decrease and the effective stresses increase, the creep process is resumed.

For a major part of the consolidation process, the creep deformation in the soil is thus a self-restrained process which cannot develop faster than permitted by the permeability of the soil and the drainage paths. The effects of the time dependence of the compressibility are that the excess pore pressures decrease more slowly with time, the rate of settlement becomes higher and the settlements never stop completely unless an unloading is performed. However, with time the rate of settlement will become so low that its practical importance diminishes.

Course of the calculations

In practice, the calculations are performed in accordance with the flowchart in *Fig. 34*. The variations in the soil properties with depth are described together with the pore pressure profile. With consideration to the variation in properties, the soil in the profile is then automatically divided into sub-layers with given properties and with thicknesses matched to facilitate a continuous calculation of the consolidation process in the profile in its entirety.

Thereafter, the load is described in terms of load intensities, load steps and time for load application. (The elastic deformations at application of the load are calculated first and after a possible adjustment of

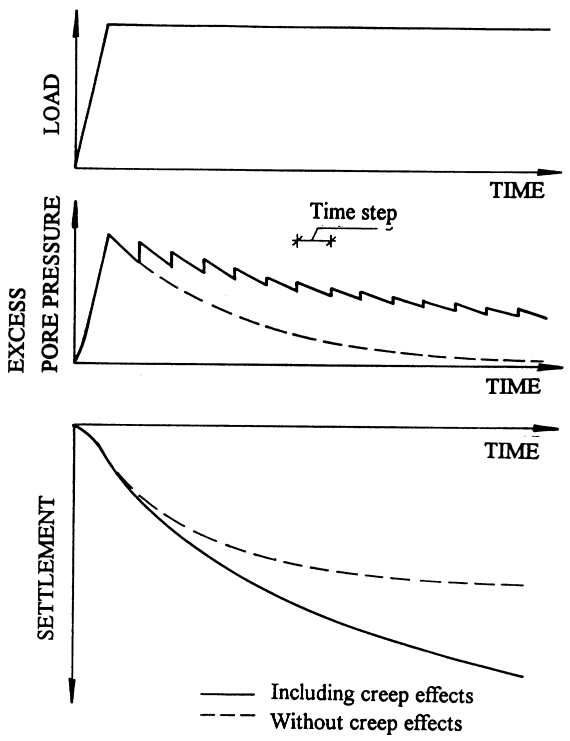


Fig. 32. Schematic presentation of calculated courses of consolidation.

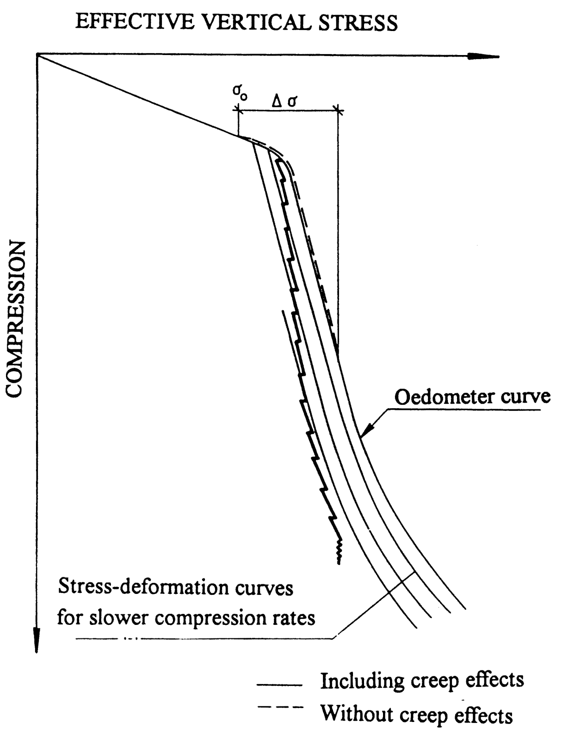


Fig. 33. Schematic presentation of calculated stress-strain relations

load and geometry the stress distribution and the excess pore pressures are calculated. The elastic deformations are calculated for the load step in its entirety and are then distributed over the time for load application.)*

(When gas is present in the soil, the initial compression of the gas bubbles is first calculated together with the corresponding initial settlement and pore pressure reduction.)*

The calculations of the consolidation process are then made for very short time steps and on the assumption that the parameters remain constant during the time step, i.e. according to the simple consolidation theory

$$\frac{\partial u}{\partial t} = -\frac{M}{g \cdot \rho_w} \cdot \frac{\partial}{\partial z}(k \frac{\partial u}{\partial z})$$

The equation is solved by using finite differences and the calculations are made with consideration to the demand for continuity in the water flow across the interfaces between the sub-layers, i.e. that according to Darcy´s law

$$k_n (\frac{\partial u}{\partial z})_n = k_{n+1} (\frac{\partial u}{\partial z})_{n+1}$$

After the calculations for the short time step, the calculated pore pressure dissipation is corrected for possible effects of gas bubbles and the settlement is calculated. The rate of compression is then calculated and compared to the reference rate, and the pore pressure is adjusted for the creep process during the time step. The thicknesses of the layers and the properties of the soil are modified with regard to the calculated compression.

Stresses and pore pressures are adjusted for possible changes in total applied load during the time step and in case the settlements have entailed that the effective load has been reduced because of a high ground water level. (When the soil contains gas, the calculated pore pressure changes because of creep effects are adjusted accordingly and the corresponding initial deformations because of the compression of the gas bubbles are calculated)*. Thereafter, the consolidation process in the following short time step is calculated, and so on.

**) Operations within parenthesis are not included in the EMBANKCO programme version 1. In this programme, the soil is assumed to be fully saturated. Elastic deformations are calculated separately when required.*

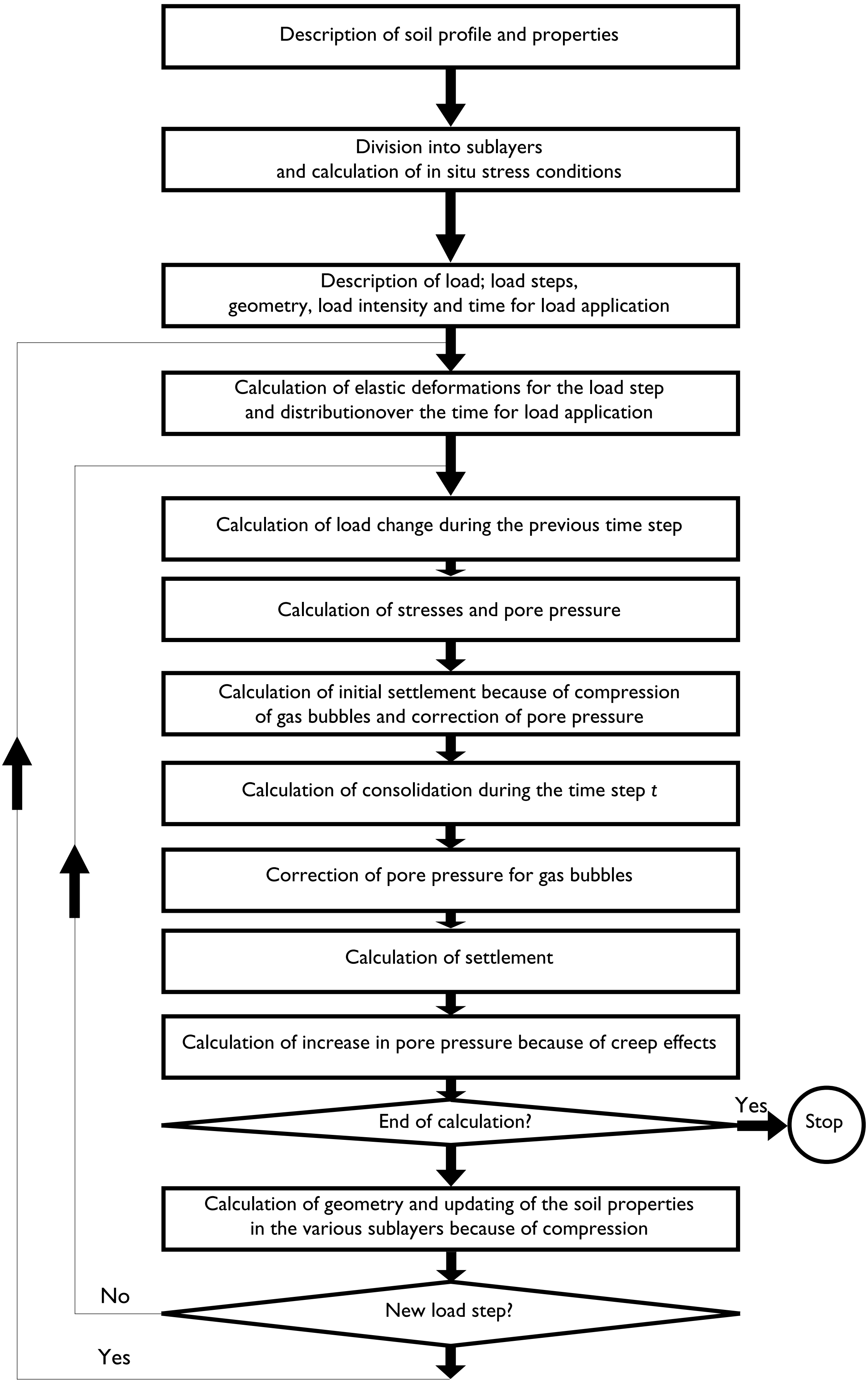


Fig. 34. Flowchart for the calculations in a settlement prediction [7].

During the course of the calculations, a register has to be kept of the stress and deformation history in the various layers and the rate effects that have been taken into account. This is required for control of the creep process. The stress and deformation history is also used for updating the apparent preconsolidation pressure and the unloading and reloading moduli.

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